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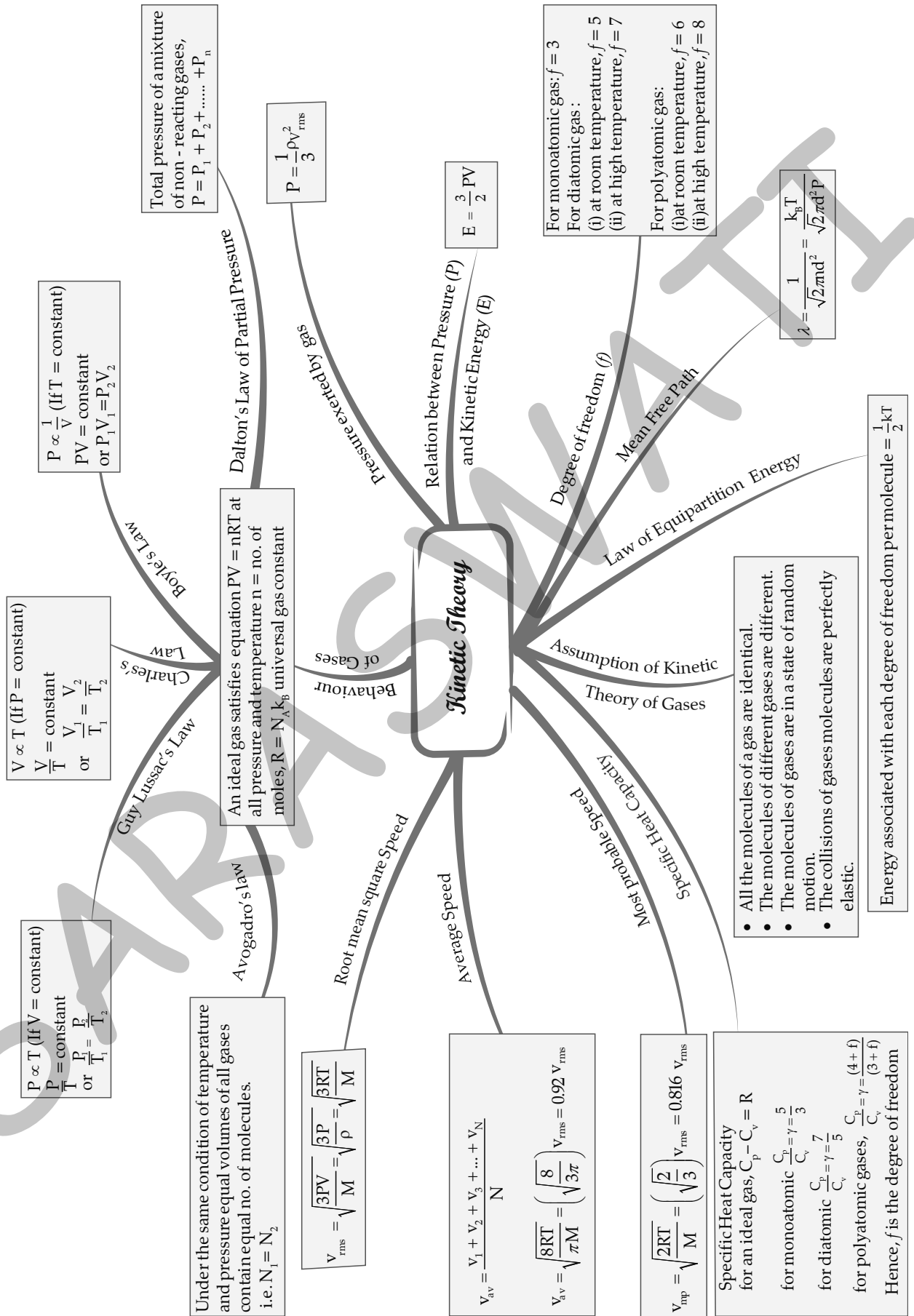
Kinetic Theory

Kinetic Theory

Equation of state of a perfect gas, work done in compressing a gas.

Kinetic theory of gases - assumptions, concept of pressure. Kinetic interpretation of temperature; rms speed of gas molecules; degrees of freedom, law of equi-partition of energy (statement only) and application to specific heat capacities of gases; concept of mean free path, Avogadro's number.

MIND MAP : LEARNING MADE SIMPLE



Introduction

What is Kinetic Theory?

- Kinetic theory explains the behaviour of gases based on the idea that the gas consists of rapidly moving atoms or molecules.
- In solids the molecules are very tightly packed as inter molecular space is not present In liquids inter molecular spaces are more as compared to solids and in gases the molecules are very loosely packed as intermolecular spaces are very large.
- The random movement of molecules in a gas is explained by kinetic theory of gases.
- We will also see that why kinetic theory is accepted as a success theory.
- Kinetic theory explains the following:-
 - a) Molecular interpretation of pressure and temperature can be explained.
 - b) It is consistent with gas laws and Avogadro's hypothesis.
 - c) Correctly explains specific heat capacities of many gases.

13.2 Molecular nature of matter

- Atomic hypothesis was given by many scientists. According to which everything in this universe is made up of atoms.
- Atoms are little particles that move around in a perpetual order attracting each other when they are little distance apart. But if they are forced very close to each other then they repel.
- **Dalton's atomic theory** is also referred as the molecular theory of matter. This theory proves that matter is made up of molecules which in turn are made up of atoms.
- According to **Gay Lussac's law** when gases combine chemically to yield another gas, their volumes are in ratios of small integers.
- **Avogadro's law** states that the equal volumes of all gases at equal temperature and pressure have the same number of molecules.
- Conclusion: - All these laws proved the molecular nature of gases.
- Dalton's molecular theory forms the basis of Kinetic theory.

Why was Dalton's theory a success?

- Matter is made up of molecules, which in turn are made up of atoms.
- Atomic structure can be viewed by an electron microscope.

Solids, Liquids, Gases in terms of molecular structure

Basis of difference	Solids	Liquids	Gases
Inter Atomic Distance(distance between molecules).	Molecules are very tightly packed. Inter atomic distance is minimum.	Molecules are not so tightly packed. Inter atomic distance is more as compared to solids.	Molecules are loosely packed .Free to move. Inter atomic distance is maximum.
Mean Free Path is the average distance a molecule can travel without colliding.	No mean free path.	Less mean free path.	There is mean free path followed by the molecules.

13.3 Behaviour of Gases

- Gases at low pressures and high temperatures much above that at which they liquify (or solidify) approximately satisfy a relation between their pressure, temperature and volume:

$$PV=KT \dots \dots \dots (i)$$

This is the universal relation which is satisfied by all gases.

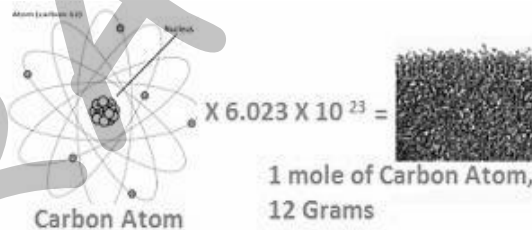
where P, V, T are pressure, volume and temperature respectively, and K is the constant for a given volume of gas. It varies with volume of gas.

$K=Nk_B$ where, N=number of molecules and k_B = Boltzmann Constant and its value never change.

- From equation (i) $PV= Nk_B$
Therefore $PV/NT = \text{constant}=(k_B)$ (Same for all gases).
- Consider there are 2 gases :- (P_1, V_1, T_1) and (P_2, V_2, T_2) where P, V and T are pressure, volume and temperature resp.
- Therefore $P_1 V_1 / (N_1 T_1) = P_2 V_2 / (N_2 T_2)$
- Conclusion: - This relation is satisfied by all gases at low pressure and high temperature.

Justification of the Avogadro's hypothesis from equation of gas

- Avogadro's hypothesis states that equal volumes of all gases at equal temperature and pressure have the same number of molecules.
- Consider the equation $PV/NT = \text{constant}$ and if P,V and T are same for 2 gases then N(number of molecules) is also same.
- According to Avogadro's hypothesis number of molecules per unit volume is same for all gases at a fixed P and T.
Avogadro number is denoted by N_A . Where, $N_A = 6.02 \times 10^{23}$. It is universal value.
- Experimentally it has been found that the mass of 24.4 litres of any gas is equal to molecular weight in grams at standard temperature and pressure.



Perfect Gas Equation

- Perfect gas equation is given by $PV = \mu RT$,
Where P,V are pressure, volume, T=absolute temperature, μ = number of moles and R =universal gas constant, $R= k_B N_A$ where, k_B = Boltzmann constant and N_A = Avogadro's Number
- This equation tells about the behaviour of gas at a particular situation.
- If a gas satisfies this equation then the gas is known as Perfect gas or an ideal gas.

Different Forms of Perfect Gas Equation

1. $PV=\mu RT \dots \dots \dots (i)$

Where μ (no. of moles) = N/N_A where N=no of molecules and N_A = Avogadro number(no of molecules in 1 mole of gas). Or $\mu = M/M_o$ where M=mass of sample of gas and M_o = molar mass.

$$PV = (N/N_A)RT \quad (\text{putting } \mu=N/N_A \text{ in equation (i)})$$

$$\text{By simplifying } PV = Nk_B T$$

$$PV=Nk_B T \Rightarrow P = (N/V) k_B T \Rightarrow P=nk_B T$$

Where, n(number density) = N/V . where, N=number of molecules and V=volume.

$$\therefore PV = nk_B T$$

2. Substitute $\mu = M/M_0$ in equation(i)

$$PV = (M/M_0) RT \Rightarrow P = (M/V) RT / M_0 \text{ where } M/V = \rho \text{ (mass density of the gas)}$$

$$\therefore P = \rho RT / M_0$$

Ideal Gas

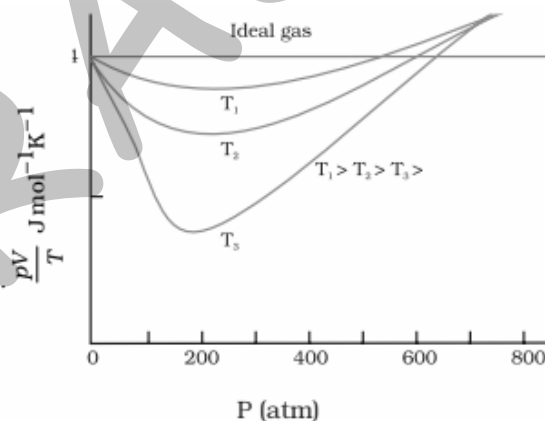
- A gas that satisfies the perfect gas equation exactly at all pressures and temperatures.
- Ideal gas is a theoretical concept.
- No real gas is truly ideal. A gas which is ideal is known as real gas.
- Real gases approach the ideal gas behaviour for low pressures and high temperatures.

Real gases deviation from ideal gas

- Real gases approach the ideal gas behaviour for low pressures and high temperatures.
- Ideal gas equation $PV = \mu RT$, for 1 mole, $\mu = 1, PV = RT$
 $\Rightarrow PV/RT = \text{constant}$

Graph should be a straight line (parallel to x-axis) for ideal gas. This means it has constant value at all temperature and all pressure.

- But in case of real gases graph approach ideal gas behaviour at high temperature and low pressure.
- At high temperature and low pressure molecules are far apart. When temperature is increased the molecules will move randomly far from each other.
- As a result molecular interaction decreases the gas behaves as an ideal gas.
- The ideal behaviour comes into picture when the molecular present inside the gas don't interact with each other.



Real gases approach ideal gas behaviour at low pressures and high temperatures.

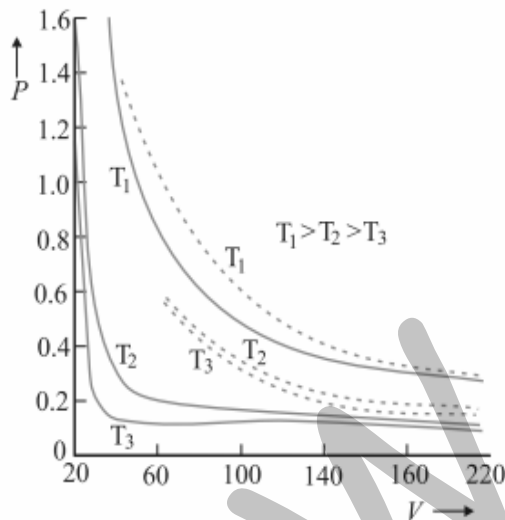
Deduction of Boyle's law and Charles law from perfect gas equation

1. Boyle's law: -Deriving Boyle's law from perfect gas equation, $PV = \mu RT$

- Consider T (temperature) and μ (no. of moles) constant.
- Therefore **PV=constant**.
- According to Boyle's law, at a constant temperature, pressure of a given mass of gas varies inversely with volume.

2. Charles's law:-Consider If P(Pressure) is constant, then

- From Perfect gas equation $PV = \mu RT, \Rightarrow V/T = \mu R/P = \text{constant}$
- Therefore $V/T = \text{constant}$.
- According to Charles's law for a fixed pressure, volume of a gas is \propto to its absolute temperature.
- Conclusion: - Ideal gas satisfies the Boyle's law and Charles's law.



Experimental P-V curves (solid lines) for steam at three temperatures compared with Boyle's law (dotted lines). P is in units of 22 atm and V in units of 0.09 litres.

Deducing Dalton's Law of partial pressures

- Dalton's law of partial pressure states that the total pressure of a mixture of ideal gases is the sum of partial pressures.
- Consider if there are several ideal gases mixed together in a vessel, then the total pressure of that vessel is equal to sum of partial pressure.
- Partial pressure is the pressure exerted by a particular gas if only that gas is present in the vessel.
- For example: -

Consider if in a vessel there is a mixture of 3 gases, A, B and C. So the partial pressure of A is equal to pressure exerted only by A and considering B and C are not present.

Similarly partial pressure of B is equal to the pressure exerted only by B and considering A and C are not there.

Similarly for C.

According to Dalton's law the total pressure of mixture is sum of partial pressure of A, partial pressure of B and partial pressure of C.

To show how perfect gas equation concludes Dalton's law of partial pressure:-

- Suppose there is a mixture of ideal gases which means these gases do not interact with each other.

By perfect gas equation, $PV = \mu RT$

Where V=volume of vessel, P=Pressure and T=temperature and μ (no. of moles).

As there are mixture of gases therefore $\mu = \mu_1 + \mu_2 + \dots$ so on.

$$PV = (\mu_1 + \mu_2 + \dots)RT$$

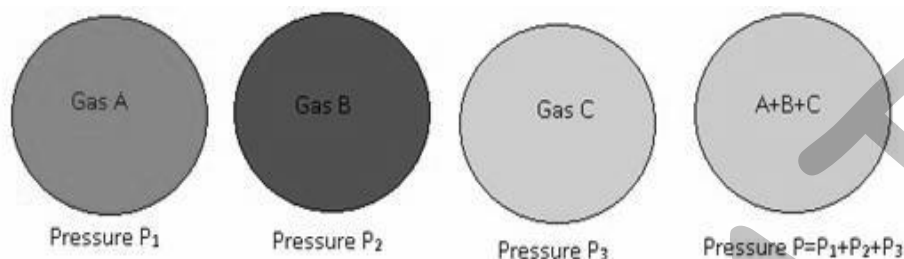
$$P = (\mu_1 + \mu_2 + \dots)RT/V$$

$$\Rightarrow P = \mu_1 RT/V + \mu_2 RT/V + \dots$$

$$\therefore P = P_1 + P_2 + \dots$$

Where P_1 = partial pressure of gas 1 and P_2 = partial pressure of gas 2.

$\therefore P = P_1 + P_2 + \dots$ total pressure due to the mixture of gases is equal to the sum of the partial pressure of the gas.



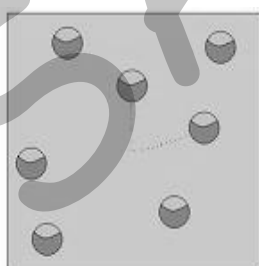
13.4 Kinetic Theory of an Ideal Gas

Basis of Kinetic Theory: -

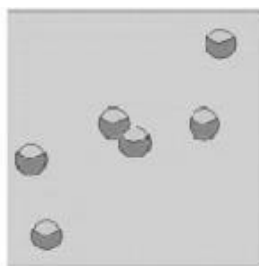
1. Molecules of gas are in incessant random motion, colliding against one another and with the walls of the container.
2. All collisions are elastic.
3. Total Kinetic energy is conserved.
4. Total momentum is conserved.
5. In case of an elastic collision total Kinetic energy and momentum before collision is equal to the total Kinetic energy and momentum after collision.

What does Kinetic Theory tells?

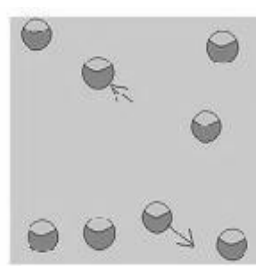
1. At ordinary temperature and pressure the molecular size is very small as compared to inter molecular distance between them.
2. In case of gas, molecules are very far from each other and the size of molecules is small as compared to the distance between them.
3. As a result, interaction between them is negligible. As there is no interaction between the molecules, there will be no force between the molecules.
4. As a result molecules are moving freely as per newton's first law of motion.
5. The molecules should move along straight line but when they come closer they experience the intermolecular forces and as a result their velocities change.
6. This phenomenon is known as collision. These collisions are elastic.



Molecules moving randomly



Molecules colliding with each other



Molecules change their direction after colliding

13.4.1 Pressure of an ideal gas based on Kinetic theory

Assumptions:-

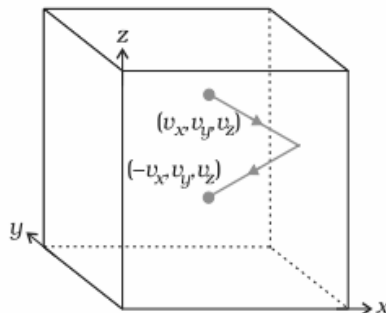
- Consider a cube shape container filled with an ideal gas. We will consider only one molecule; the molecule hits the walls of the container and bounces back.
- Let the velocity of the molecule when it is moving be (v_x, v_y, v_z) .
- When the molecule bounces back, the velocity will be $(-v_x, v_y, v_z)$.
- Change in momentum = $P_f - P_i$ where P_f = final momentum and P_i = initial momentum)
- $P_f - P_i = -mv_x - mv_x = -2mv_x$
- This change in momentum is imparted to the wall due to the collision.
- Momentum imparted to the wall in collision by one molecule = $2mv_x$
 - But there are as many molecules, we have to calculate total momentum imparted to the wall by all of them.
 - To calculate the number of molecules that hit the wall:
Area of wall = A
 \therefore in time Δt , all molecules within a distance of $Av_x\Delta t$ can hit the wall.
but on an average half of molecules move towards the wall and half away from the wall.
 $\therefore \left(\frac{1}{2}\right) Av_x\Delta t$ will hit the wall.
- \therefore total momentum imparted to the wall = $2mv_x \times \frac{1}{2} n Av_x\Delta t = Anv_x^2\Delta tm$
- Force exerted on the wall = rate of change of momentum = Anv_x^2m
- Pressure on the wall $P = \frac{F}{A} = nmv_x^2$
- $\therefore P = nmv_x^2$ is true for group of molecules moving with velocity v_x

Note:

- i. All the molecules inside the gas will not have the same value of velocity. All will have different velocities
 - ii. The above equation therefore, is valid for pressure due to the group of molecules with speed v_x in the x-direction and n stands for the number density of that group of molecules.
 - Therefore total pressure due to all such groups will be obtained by summing over the contribution due to the molecule $P = nm\overline{v_x^2}$
 - Where, $\overline{v_x^2}$ is the average of v_x^2 .
- Since the gas is isotropic the molecules move randomly which means the velocity of all the molecules can be in any direction. Therefore

$$\overline{v_x^2} = \overline{v_y^2} = \overline{v_z^2} = \frac{1}{3}(\overline{v_x^2} + \overline{v_y^2} + \overline{v_z^2}) = \frac{1}{3}v^2$$

- Therefore, Pressure, $P = \frac{1}{3}nmv^2$, where v^2 = average squared speed.



Elastic collision of a gas molecule with the wall of the container.

Justifying the assumptions:-

- We have assumed the container containing the gas is a cube. The shape of the container is immaterial.
- For a vessel of any arbitrary shape, we can choose a small infinitesimal (planar) area and can prove the above derivation.
- We will see A and Δt are not there in the final result.
- By Pascal's law pressure in one portion of gas in equilibrium is the same as anywhere else.
- All collisions are neglected.
 - a) The number of molecules hitting the wall in time Δt was found to be $\left(\frac{1}{2}\right) Av_x \Delta t$, with random collisions and a steady state of gas.
 - b) Thus, if a molecule with velocity (v_x, v_y, v_z) acquires a different velocity due to collision with some molecules, there will always be some other molecule with a different initial velocity which after a collision acquires the velocity (v_x, v_y, v_z) .
 - c) Molecular collision, when they are not too frequent and the time spent in collision is very small compared to the time between collisions, will not have any affect in the above calculation.

13.4.2 Kinetic Interpretation of Temperature

- The average kinetic energy of a molecule is directly proportional to the absolute temperature of the gas.
- It is independent of pressure, volume or nature of the ideal gas.
- Multiplying with V on both sides,

$$PV = \frac{1}{3} nV m \overline{v^2}$$

After simplifying,

$$PV = \frac{2}{3} N \times \frac{1}{2} m \overline{v^2}, \text{ where, } n = N/V$$

N - Number of molecules in a sample.

- Therefore $PV = \frac{2}{3} E \dots \dots \dots (i)$
 E - Kinetic energy = $N \times \frac{1}{2} m \overline{v^2}$
 This is the basis of Kinetic interpretation of temperature.
- Combining eq(i) with the ideal gas equation, we get

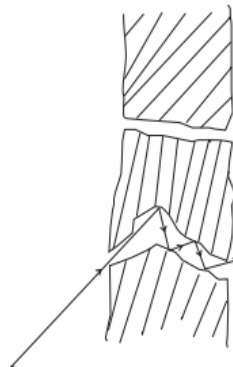
$$E = \frac{3}{2} k_B N T \dots \dots \dots (ii)$$

$$\text{or, } \frac{E}{N} = \frac{1}{2} m \overline{v^2} = \frac{3}{2} k_B T \dots \dots \dots (iii)$$

This is the average kinetic energy

$$\therefore KE \propto T$$

Hence, temperature can be interpreted as a molecular quantity.



Molecules going through a porous wall.

Kinetic theory: Consistent with ideal gas equation and gas laws

1. Consistent with ideal gas equation:

$$\text{From kinetic interpretation } \frac{E}{N} = \frac{3}{2} k_B T \Rightarrow E = \frac{3}{2} N k_B T$$

The internal energy of an ideal gas is directly proportional to T

This shows that internal energy of an ideal gas depends only on temperature, not on pressure or volume.

2. Consistent with Dalton's law of partial pressures:

$$P = \frac{1}{3} n m \bar{v}^2, \text{ from kinetic theory}$$

∴ if a vessel contains a mixture of gases,

$$P = \frac{1}{3} [n_1 m_1 \bar{v}_1^2 + n_2 m_2 \bar{v}_2^2 + \dots]$$

in equilibrium average Kinetic energy of molecules of different gasses will be equal

$$\frac{1}{2} m_1 \bar{v}_1^2 = \frac{1}{2} m_2 \bar{v}_2^2 = \dots = \frac{3}{2} k_B T$$

$$\text{Total pressure, } P = \frac{1}{3} \left[\frac{3}{2} n_1 k_B T + \frac{3}{2} n_2 k_B T + \dots \right]$$

$$P = k_B T [n_1 + n_2 + \dots]$$

$$P = \frac{R}{N_A} [n_1 + n_2 + \dots]$$

$$P = [\mu_1 + \mu_2 + \dots] RT$$

$$P = P_1 + P_2 + \dots$$

This is Dalton's law of partial pressure.

Law of Equipartition of energy: Degrees of Freedom

- Degrees of Freedom can be defined as independent displacements or rotations that specify the orientation of a body or system.
- A molecule free to move in space needs three coordinates to specify its location.
- If it is constrained to move in a plane it needs two.
- If constrained to move along a line, it needs just one coordinate to locate it.
- For example:- Consider a room and if we tie a thick rope from one wall to another.
- Take a ball which is moving straight on the rope.
- The ball has only 1 degree of freedom. It can move only in one particular dimension.
- Consider if the ball is on the floor which is two-dimensional, then the ball can move along 2 directions.
- The ball has 2 degree of freedoms.
- Consider if we throw the ball in space which is 3 dimensional. Then the ball can move in 3 dimensions.
- Therefore degree of freedom tells us in how many ways a body can move or rotate or vibrate.

Categories of Degrees of Freedom

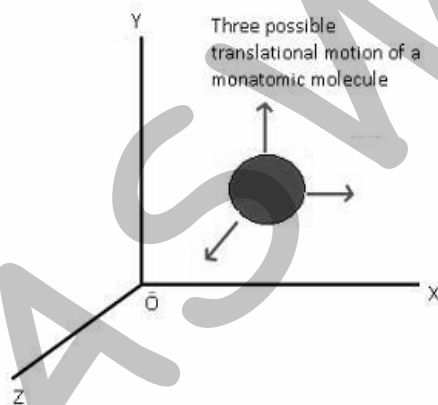
1. Translational degree of freedom.
2. Rotational degree of freedom.
3. Vibrational degree of freedom.

Translational degree of freedom:-

- Translation means motion of the body as a whole from one point to another.
- For example:
- Consider the oxygen molecule; it has 2 oxygen atoms which are bonded together.

- The 2 oxygen atoms along with the bond are considered as whole body.
- When the body as a whole is moving from one point to another is known as translational.
- Consider a molecule which is free to move in space and so it will need 3 coordinates(x, y, z) to specify its location.
- Therefore it has 3 degrees of freedom.
- Similarly a molecule which is free to move in a plane which is 2 dimensional and so it needs 2 coordinates to specify its location.
- Therefore it has 2 degrees of freedom.
- Similarly a molecule which is free to move in line it needs 1 coordinate to specify its location.
- Therefore it has 1 degree of freedom.
- Molecules of monoatomic gas have only translational degrees of freedom. This means gases which have only one atom.
- For example:-Helium atom it consists of only one He atom. It will have translational degrees of freedom.
- Each translational degree of freedom contributes a term that contains square of some variable of motion.
- The variable of motion means the velocity (v_x, v_y, v_z).
- The term $(1/2) mv_x^2$ will contribute to energy. This is Kinetic energy which is involved with the motion of the molecule from one point to another.

In thermal equilibrium, the average of each such term is $(1/2) k_B T$.



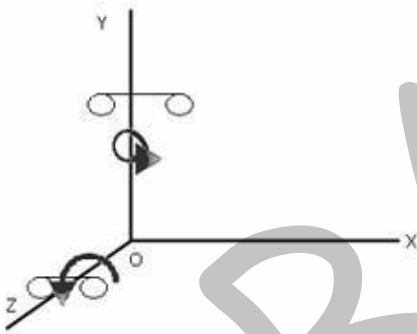
Rotational Degree of freedom

- Independent rotations that specify the orientation of a body or system.
- There is rotation of one part of the body with respect to the other part.
- Rotational degree of freedom happens only in diatomic gas.
- Diatomic molecules have rotational degrees of freedom in addition to translational degrees of freedom.
- It is possible in diatomic molecules as 2 atoms are connected together by a bond. So the rotation of one atom w.r.t to other atom.
- In diatomic there is translational in addition to that they have rotational degree of freedom also.
- For example: - Two oxygen atoms joined together by a bond. There are two perpendicular axes.
- There are 2 rotations possible along the two axes.
- They have 3 translational degrees of freedom and also 2 rotational degree of rotation.
- Therefore Rotational degree of freedom contributes a term to the energy that contains square of a rotational variable of motion.
- Rotational variable of motion comes from angular momentum ω .
- Linear velocity is v_x, v_y, v_z . Whereas angular velocity is w_x, w_y, w_z .

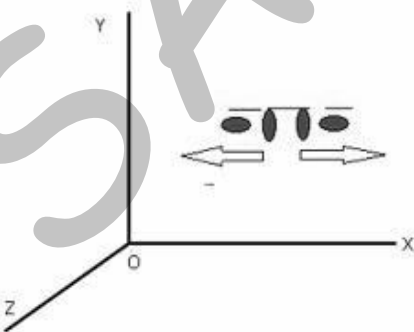
- $E_R(\text{rotational}) = (1/2)(I_1\omega_1) + (1/2)I_2\omega_2$. These are 3 rotational degrees of freedom along the 2 perpendicular axes.
- The total energy contribution due to the degrees of freedom for oxygen molecule.
- There will be 3 translational degree of freedom $(1/2)m_x v_x^2, (1/2)m_y v_y^2, (1/2)m_z v_z^2$
- 2 rotational degree of freedom $(1/2)I_1^2\omega_1^2, (1/2)I_2^2\omega_2^2$

Vibrational degree of freedom

- Some molecules have a mode of vibration, i.e. its atoms oscillate along the inter-atomic axis like a one-dimensional oscillator.
- This vibration is observed in some molecules.
- For example:- CO atoms oscillate along the interatomic axis like a one-dimensional oscillator.
- Consider two atoms they vibrate along the inter-atomic axis.
- The vibrational energy terms contain square of vibrational variables of motion.
- Total vibrational energy term $E_v = (1/2) m (dy/dt)^2 + (1/2) ky^2$ where $(1/2) m(dy/dt)^2 = \text{Kinetic energy}$ and $(1/2)ky^2 = \text{Potential energy}$ and $k = \text{force constant one-dimensional oscillator}$.
- The vibrational degree of freedom contributes 2 terms.



(1)



(2)

- (1) Rotational motion along two axis perpendicular to line joining two particles (here y and z directions)
- (2) Vibrational motion along line joining the two atoms

Comparison between 3 energy modes

Translational	Rotational	Vibrational
$E_t = (1/2)mv_x^2$	$E_r = (1/2)I\omega^2$	$E_v = (1/2)m(dy/dt)^2 + (1/2)ky^2$
1 squared term is being contributed	1 squared term is being contributed	2 squared term is being contributed

Law of Equipartition of energy

According to this law, in equilibrium, the total energy is equally distributed in all possible energy modes, with each mode having an average energy equal to $(1/2)k_B T$.

1. Each translational degree of freedom contributes $(1/2)k_B$
2. Each rotational degree of freedom contributes $(1/2)k_B$
3. Each vibrational degree of freedom contributes $2 \times (1/2)k_B$

Specific Heat Capacity for monoatomic gases

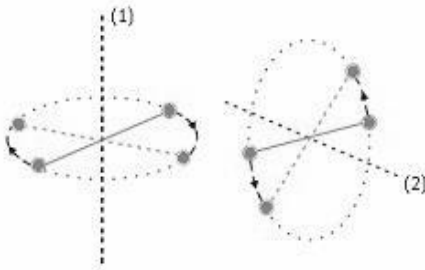
- Monoatomic gases will only have translational degree of freedom.
- Maximum they can have is three translational degrees of freedom.
- Each degree of freedom will contribute $(1/2)k_B$
- Therefore 3 degrees of freedom will contribute $(3/2)k_B$
- By using law of equipartition of energy, the total internal energy of 1 mole of gas $U = (3/2)k_B T \times N_A = (3/2)RT$
- Specific heat capacity at constant volume $C_v = dU/dT = (3/2)R$ (i)
- For an ideal gas $C_p - C_v = R$, By using equation(i) $C_p = (5/2)R$
- Ratio of specific heats $\gamma = C_p/C_v = (5/3)$

Specific Heat of Diatomic gases (rigid)

- A rigid diatomic gas means they will have translational as well as rotational degree of freedom but not vibrational.
- They are rigid oscillator.
- A rigid diatomic molecule will have 3 translational degrees of freedom and 2 rotational degrees of freedom. Total 5 degrees of freedom.
- By law of equipartition of energy, each degree of freedom will contribute $(1/2)k_B$
- Therefore 5 degree of freedom will contribute $(5/2)k_B$
- Therefore the total internal energy of 1 mole of gas, $U = (5/2)k_B T \times N_A = (5/2)RT$
- Specific heat capacity at constant volume (C_v) $= dU/dT = (5/2)R$
- Specific heat capacity at constant pressure of a rigid diatomic is given as $C_p = (7/2)R$
- Ratio of specific heats $\gamma = C_p/C_v = (7/5)$

Specific Heat of Diatomic gases (non-rigid)

- A non-rigid diatomic gas has translational, rotational as well as vibrational degrees of freedom.
- There will be 3 translational degrees of freedom and 2 rotational degrees of freedom and 1 vibrational degree of freedom.
- Total contribution by translational = $(1/2) k_B T$, rotational = $2 \times (1/2) k_B T$ and vibrational = $k_B T$
- Total Internal energy for 1 mole = $(5/2) k_B T + k_B T = (7/2) k_B T = (7/2) RT$.
- $C_V = dU/dT = (7/2) R$.
- $C_P = C_V + R = (9/2) R$.
- $\gamma = C_P/C_V = (9/7)$



There are two independent axes of rotation (1) and (2) normal to the axis joining the two oxygen molecules. It has 3 translational and 2 rotational degrees of freedom.

Specific Heat Capacity for polyatomic gases

- Polyatomic gases will have 3 translational degrees of freedom, 3 rotational degrees of freedom and 'f' number of vibrational modes.
- Total internal energy of 1 mole of gas = $(3 \times (1/2) k_B T + 3 \times (1/2) k_B T + f k_B T) \times N_A$
 $((3/2) + (3/2) + f) RT = (3+f) RT$.
- $C_V = dU/dT = (3+f) R$
- $C_P = C_V + R = (4+f) R$
- $\gamma = C_P/C_V = (4+f)/(3+f)$

Specific Heat Capacity for solids

- Consider there are N atoms in a solid. Each atom can oscillate about its mean position.
- Therefore vibrational degree of freedom = $k_B T$
- In one-dimensional average energy = $k_B T$, in three-dimensional average energy = $3 k_B T$
- Therefore total internal energy (U) of 1 mole of solid = $3 k_B T \times N_A = 3RT$
- At constant pressure, $\Delta Q = \Delta U + P \Delta V$ change in volume is very less in solids. Therefore $\Delta V = 0$.
- $\Rightarrow \Delta Q = \Delta U$
- $C_V = (dU/dT)_V$
- $C_P = (dQ/dT)_P$ as $\Delta Q = \Delta U$, Therefore $C_V = dU/dT = 3R$
- Therefore $C_P = C_V = 3R$

Specific Heat Capacity of water

- Consider water as solid, so it will have 'N' number of atoms.
- Therefore for each atom average energy $=3k_B T$
- No of molecules in $H_2O = 3$ atoms.
- Total internal energy $U = 3k_B T \times 3 \times N_A = 9RT$.
- $C_V = C_P = 9R$.

Conclusion on Specific heat

- According to classical mechanics, the specific heat which is calculated based on degree of freedom should be independent of temperature.
- However $T \rightarrow 0$, degree of freedom becomes inefficient.
- This shows classical mechanics is not enough; as a result quantum mechanics came into play.
- According to quantum mechanics minimum non-zero energy is required for degree of freedom to come into play.
- Specific heats of all substances approach zero as $T \rightarrow 0$.

Mean free path

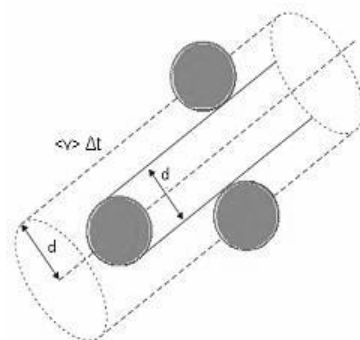
- Mean free path is the average distance between the two successive collisions.
- Inside the gas there are several molecules which are randomly moving and colliding with each other.
- The distance which a particular gas molecule travels without colliding is known as mean free path.

Expression for mean free path

- Consider each molecule of gas is a sphere of diameter (d). The average speed of each molecule is $\langle v \rangle$.
- Suppose the molecule suffers collision with any other molecule within the distance (d). Any molecule which comes within the distance range of its diameter this molecule will have collision with that molecule.
- The volume within which a molecule suffer collision $= \langle v \rangle \Delta t \pi d^2$.
- Let number of molecules per unit volume $= n$
- Therefore the total number of collisions in time $\Delta t = \langle v \rangle \Delta t \pi d^2 n$
- Rate of collision $= \langle v \rangle \Delta t \pi d^2 n / \Delta t = \langle v \rangle \pi d^2 n$
- Suppose time between collision $\tau = 1 / \langle v \rangle \pi d^2 n$
- Average distance between collision $= \tau \langle v \rangle = 1 / \pi d^2 n$
- $1 / \pi d^2 n$ this value was modified and a factor was introduced.
- Mean free path (l) $= \frac{1}{\sqrt{2} \pi d^2 n}$

Conclusion: - Mean free path depends inversely on:

- a) Number density (number of molecules per unit volume)
- b) Size of the molecule.



The volume swept by a molecule in time Δt in which any molecule will collide with it.

QUESTION ALIKE

Very Short Answer Type Questions :

1. Can we increase the temperature of a gas keeping its pressure and volume constant?

Sol. No, temperature cannot be changed without changing pressure and volume.

2. The absolute temperature of a gas is increased 4 times. What will be the change in the rms velocity of the gas molecules?

Sol. $v_{\text{rms}} \propto \sqrt{T}$

New rms velocity is $2v_{\text{rms}}$

So change = $2v_{\text{rms}} - v_{\text{rms}} = v_{\text{rms}}$

3. What will be the ratio of translational kinetic energy of hydrogen and nitrogen molecules at the same temperature?

Sol. One, because the kinetic energy per molecule of the gas depends only upon the temperature.

4. What will be the ratio of the rms velocities of the molecules of two gases A and B if their vapour densities are in the ratio 9 : 16?

Sol. $\frac{(v_{\text{rms}})_1}{(v_{\text{rms}})_2} = \sqrt{\frac{\rho_2}{\rho_1}} = \sqrt{\frac{9}{16}} = 3 : 4$

5. Calculate the root mean square velocity of a given sample having three molecules having velocity 1, 2 and 3 m/s.

Sol. $v_{\text{rms}} = \sqrt{\frac{v_1^2 + v_2^2 + v_3^2}{3}} = \sqrt{\frac{1 + 4 + 9}{3}} = \sqrt{\frac{14}{3}}$ m/s

6. Define degree of freedom.

Sol. The term degree of freedom of a system refers to the possible independent motions a system can have or number of possible independent ways in which system can have energy.

7. Give the dependence of mean free path of molecules of a gas on number density and size of molecule.

Sol. Mean free path (l) is given by

$$l = \frac{1}{\sqrt{2}n\pi d^2}$$

where n is no. density and d is the diameter of the gas molecule.

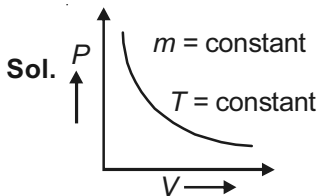
8. If the temperature of a gas increases, what will happen to its mean square speed?

Sol. If the temperature increases, the mean square speed of gas molecules will increase as $\bar{v}^2 \propto T$.

9. How many degrees of freedom a diatomic gas molecule can have?

Sol. A diatomic gas have 5 degree of freedom, 3 translational and 2 rotational. If diatomic molecule is not rigid then due to vibratory motion 2 degree of freedom will be added. *i.e.*, 7 degree of freedom.

10. If temperature and mass of a gas are kept constant, draw variation of P (pressure) with V (volume).



Short Answer Type Questions :

11. On the basis of kinetic theory of gases, explain how the temperature of a gas rises on heating.

Sol. When heat is given to a gas, the rms velocity of the gas molecules increases. As $v_{\text{rms}} \propto \sqrt{T}$, so the temperature of the gas increases.

12. On increasing the temperature of a gas filled in a cylinder, the pressure of the gas increases. Explain.

Sol. On raising the temperature, the average velocity of the gas molecules increases. As a result of which more and more molecules collide with the wall of the cylinder per second. Hence, greater momentum is transferred to the wall per second and the pressure increases.

13. Explain, on the basis of kinetic theory, how the pressure of a gas changes if its volume is reduced at constant temperature.

Sol. On reducing the volume, the space for the given number of molecules of the gas decreases *i.e.*, number of molecules per unit volume increases. As a result of which more molecules collide with the walls of the container per second transferred more momentum to the walls. Hence, the pressure increases.

14. Distinguish between average speed and rms speed.

Sol. Average speed is the arithmetic mean of the speeds of the molecules where as rms speed is the root mean square speed *i.e.*, the square root of the mean of the squares of different speeds of the individual molecules.

15. State Boyle's law.

Sol. According to Boyle's law, when the temperature of a certain mass of a gas is kept constant, the volume V occupied by the gas is inversely proportional to the pressure P exerted by the gas *i.e.*,

$$\text{when } T \text{ is constant, } V \propto \frac{1}{P} \text{ or } V = \frac{\text{constant}}{P}$$

$$PV = \text{constant}$$

16. State Charles's law.

Sol. According to this law, when pressure P of a certain mass of a gas is kept constant, the volume V occupied by the gas is directly proportional to the temperature T of the gas, *i.e.*, when P is constant

$$V \propto T$$

$$\frac{V}{T} = \text{constant}$$

17. Two vessels A and B are filled with the same gas, where volume, temperature and pressure in A is twice the volume, temperature and pressure in B . Calculate the ratio of the number of molecules of gas in A and B .

Sol. $PV = nRT$

$$\Rightarrow n = \frac{PV}{RT}, \text{ number of molecules} = n_A N$$

$$\frac{P_A V_A}{T_A} = n_A R, \quad \frac{P_B V_B}{T_B} = n_B R$$

$$\frac{n_A}{n_B} = \frac{P_A V_A}{T_A} \times \frac{T_B}{P_B V_B} = \frac{2P \times 2V}{2T} \times \frac{T}{PV} = \frac{2}{1} = 2:1$$

18. On what factor the average kinetic energy of gas molecules depend? What will the kinetic energy of the molecules at the absolute zero?

Sol. The average kinetic energy of gas molecules depends upon the temperature of the gas. At absolute zero, rms velocity of the gas molecules becomes zero so mean K.E. per molecules of the gas becomes zero at absolute zero.

19. A gas in a cylinder is at pressure P . If the masses of all the molecules be made one-third and their speeds be made doubled, then find the resultant pressure.

Sol. $P = \frac{1}{3} \frac{mnv_{\text{rms}}^2}{V}$

$$m' = \frac{m}{3}, \quad v'_{\text{rms}} = 2v_{\text{rms}}$$

$$P' = \frac{1}{3} \times \frac{\left(\frac{m}{3}\right) n \times (2v_{\text{rms}})^2}{V} = \frac{4P}{3}$$

$$\boxed{P' = \frac{4}{3}P}$$

20. A vessel contains a gas at pressure P_0 . Show that the pressure becomes double, if the masses of all the molecules are halved and their speeds doubled.

Sol. $P = \frac{1}{3} \frac{mn}{V} v_{\text{rms}}^2$

m is halved and v_{rms} is doubled. Hence, P will become two times.

$$P' = \frac{1}{3} \times \frac{\left(\frac{1}{2}m\right) n \times (2v_{\text{rms}})^2}{V}$$

$$\boxed{P' = 2P_0}$$

Hence proved.

21. The root mean square velocity of the gas molecules is 400 m/s. If the atomic weight is doubled and absolute temperature is halved then, calculate the root mean square speed of the gas molecules.

Sol. $v_{\text{rms}} = \sqrt{\frac{3RT}{M}}$

$$v'_{\text{rms}} = \sqrt{\frac{3RT'}{M'}}$$

$$T' = \frac{T}{2}, M' = 2M$$

$$\therefore v'_{\text{rms}} = \sqrt{\frac{3RT}{4M}} = \frac{1}{2} v_{\text{rms}} = \frac{1}{2} \times 400 = 200 \text{ m/s}$$

22. A vessel contain N molecules of a gas at absolute temperature T . If the number of molecules is halved, what should be the absolute temperature so as the kinetic energy of the molecules remains the same.

Sol. K.E. = $\frac{3}{2} nkT$

when the number of molecules is halved. K.E. becomes double so, to keep the K.E. same temperature should be doubled.

23. Show that the r.m.s. velocity of oxygen is $\sqrt{2}$ times that of sulphur dioxide.

Sol. $\frac{(v_{\text{r.m.s.}})_{\text{O}_2}}{(v_{\text{r.m.s.}})_{\text{SO}_2}} = \sqrt{\frac{M_{\text{SO}_2}}{M_{\text{O}_2}}}$ Atomic weight of sulphur is 32 and that of oxygen is 16.

$$= \sqrt{\frac{32 + 32}{32}}$$

$$= \sqrt{2}$$

or $(v_{\text{r.m.s.}})_{\text{O}_2} = \sqrt{2} (v_{\text{r.m.s.}})_{\text{SO}_2}$

24. Calculate the average kinetic energy of a oxygen molecule (in ergs), if it occupies a volume of 2×10^4 cc at a pressure of 10^5 dyne/cm². (use $N_0 = 6 \times 10^{23}$)

Sol. Average KE = $\frac{1}{2} m v_{\text{rms}}^2$

$$= \frac{1}{2} m \cdot \frac{3PV}{M}$$

$$= \frac{1}{2} m \cdot \frac{3PV}{mN_A} = \frac{1}{2} \frac{3PV}{N_A}$$

$$= \frac{1}{2} \times \frac{3 \times 10^5 \times 2 \times 10^4}{6.023 \times 10^{23}}$$

$$= 0.5 \times 10^{-14} \text{ erg.}$$

25. The average kinetic energy of a gas molecule at 27°C is 3.3×10^{-21} J. Calculate its average kinetic energy at 227°C .

Sol. $\text{K.E.} = \frac{3}{2} k_B T$

at $27^\circ\text{C} \Rightarrow T = 300$ K

at $227^\circ\text{C} \Rightarrow T = 500$ K

$$\frac{\text{KE}_1}{\text{KE}_2} = \frac{T_1}{T_2}$$

$$\text{KE}_2 = \frac{\text{KE}_1 \times T_2}{T_1} = \frac{3.3 \times 10^{-21} \times 500}{300} = 5.5 \times 10^{-21} \text{ J}$$

26. Mention the condition when a real gas obeys ideal gas equation and why?

Sol. At low pressure and high temperature, the molecules are farther apart so that molecular size is negligible as compared to the size of the container.

27. Two perfect gases at temperature T_1 and T_2 are mixed. If there is no loss of energy and the masses and number of molecules of the gases are m_1, m_2 and n_1, n_2 respectively. Calculate the temperature of the mixture.

Sol. $\text{K.E.}_1 = n_1 \frac{3}{2} k T_1$

$$\text{K.E.}_2 = n_2 \frac{3}{2} k T_2$$

$$\text{Total K.E.} = (n_1 + n_2) \frac{3}{2} k T$$

$$\text{Total K.E.} = \text{K.E.}_1 + \text{K.E.}_2$$

$$(n_1 + n_2) \frac{3}{2} k T = n_1 \frac{3}{2} k T_1 + n_2 \frac{3}{2} k T_2$$

$$T = \frac{n_1 T_1 + n_2 T_2}{n_1 + n_2}$$

28. A gas is filled in a cylinder fitted with a piston at a definite temperature and pressure. Explain on the basis of kinetic theory why on pulling the piston out; the pressure of gas decreases.

Sol. On pulling the piston out, the volume of gas increases. Due to this, less number of molecules collide with the walls of cylinder per second and hence less momentum is transferred to the walls per second. Also, area of the walls on which these collisions take place increases. Due, to this pressure decreases.

29. Two vessels *A* and *B* are identical. *A* has 1 gm of hydrogen at 0°C and *B* has 1 gm oxygen at 0°C .

- Which vessel does contain more molecules and how much?
- In which vessel the pressure of the gas is higher and how much?

Sol. (i) Vessel *A* contains 16 times more (hydrogen) molecules contained in *B*, because the number of molecules in 1 gm hydrogen is 16 times the number of molecules in 1 gm oxygen.

- At a given temperature, the average kinetic energy per molecule $\left(\frac{3}{2} k T\right)$ is independent of the nature of the gas. Now, from the gas equation $PV = nKT$ for n molecules.

Thus, for a given volume, the gas pressure is proportional to the number of molecules. Hence, pressure in *A* will be 16 times higher than that in *B*.

30. State law of equipartition of energy and hence calculate molar specific heats of mono, di and triatomic gases at volume and constant pressure.

Sol. According to law of equipartition of energy $\frac{1}{2} k_B T$ energy is associated with every degree of freedom of a gas molecule, where k_B is Boltzmann constant and T is temperature of the system.

(a) Specific heat capacity of monoatomic gases

In case of mono-atomic gases, a molecule has 3 degrees of freedom.

Average energy associated with three degrees of freedom = $\frac{3}{2} k_B T$.

The total energy of one gram mole of the monoatomic gas $U = \frac{3}{2} k_B T \times N_A = \frac{3}{2} RT$

but $C_v = \frac{dU}{dT}$. Hence $C_v = \frac{d}{dT} \left(\frac{3}{2} RT \right) = \frac{3}{2} R$.

We know that $C_p - C_v = R$

Hence $C_p = R + C_v = \frac{3}{2} R + R = \frac{5}{2} R$

- (b)** In case of diatomic gases, a molecule is treated like a rigid rotator. It has 5 degrees of freedom, 3 translational and 2 rotational.

Similarly as in case of mono-atomic gases

$U = 5 \times \left(\frac{1}{2} k_B T \right) \times N_A = \frac{5}{2} RT$

$C_v = \frac{dU}{dT} = \frac{5}{2} R$

$C_p = C_v + R = \frac{5}{2} R + R = \frac{7}{2} R$

If diatomic gas is not rigid rotator, it also has one vibrational mode, hence 7 degrees of freedom. With vibrational mode, 2 degrees of freedom are associated.

Hence $U = \left(\frac{5}{2} k_B T + k_B T \right) \times N_A = \frac{7}{2} RT$

$C_v = \frac{dU}{dT} = \frac{7}{2} R$

$C_p = C_v + R = \frac{9}{2} R$.

(c) For triatomic gases

- (i) In case of linear triatomic gases, there are seven degrees of freedom

$$U = 7 \times \frac{1}{2} k_B T \times N_A = \frac{7}{2} RT$$

$$C_v = \frac{dU}{dT} = \frac{7}{2} R$$

$$C_p = C_v + R = \frac{7}{2} R + R = \frac{9}{2} R$$

- (ii) In case of non-linear triatomic gas molecules, there are six degrees of freedom,

$$\text{Hence } U = 6 \times \frac{1}{2} k_B T \times N_A = 3RT$$

$$C_v = \frac{dU}{dT} = \frac{d}{dT}(3RT) = 3R$$

$$C_p = C_v + R = 3R + R = 4R$$

Long Answer Type Questions :

31. What is an ideal gas? Under what conditions of pressure and temperature can a gas be assumed as an ideal gas?

Sol. Ideal gas is a gas which strictly obey gas laws. For such a gas, the size of the molecules of a gas is zero and there is no force of attraction or repulsion amongst its molecules. The ideal gas equation connecting pressure (P), volume (V) and absolute temperature (T) is given by

$$PV = \mu RT = k_B NT$$

where μ is the number of moles and N is the number of molecules. R and k_B are constants

$$R = 8.314 \text{ J mol}^{-1} \text{ K}^{-1}, \quad k_B = \frac{R}{N_A} = 1.38 \times 10^{-23} \text{ JK}^{-1}$$

Real gases satisfy the ideal gas equation only at low pressures and high temperatures only approximately.

32. Give the postulates of kinetic theory of gases and on its basis, find the expression for the pressure of an ideal gas.

Sol. Assumptions of kinetic theory of gases

- (i) A gas consists of a large number of identical, tiny, spherical, neutral and elastic particles called molecules.
- (ii) In a gas molecules are moving in all possible directions with all possible speeds in accordance with Maxwell's distribution law.
- (iii) The space occupied by the molecules is much smaller than the volume of the gas.
- (iv) There is no force of attraction among the molecules.
- (v) The pressure of gas is due to elastic collision of gas molecules with the walls of the container.

- (vi) The time of contact of a moving molecules with the walls of the container is negligible as compared to the time interval between two successive collisions on the same wall of the container.

Pressure P exerted by an ideal gas is given by

$$P = \frac{1}{3} \frac{mN}{V} \overline{v^2}$$

where $\overline{v^2}$ is the mean square velocity and m is the mass of each molecule.

$$\overline{v^2} = \left[\frac{v_1^2 + v_2^2 + \dots}{N} \right]$$

N is the total number of molecules in the vessel having volume V .

33. Assuming the relation $P = \frac{1}{3} \rho \overline{v^2}$ of kinetic theory, prove that the average kinetic energy of a molecule of an ideal gas is directly proportional to the absolute temperature of the gas.

Sol. The average kinetic energy of 1 gm molecule of the gas

$$= \frac{1}{2} M \overline{v^2} = \frac{1}{2} M \left(\frac{3P}{\rho} \right) \quad \left[\text{Using, } P = \frac{1}{3} \rho \overline{v^2} \right]$$

$$\Rightarrow \text{K.E.} = \frac{3}{2} M \left(\frac{RT}{\rho V} \right) = \frac{3}{2} RT \quad \left[\rho = \frac{M}{V} \right]$$

There are N molecules in 1 gram molecule of the gas. Hence the average kinetic energy of 1 molecule

$$= \frac{\frac{3}{2} RT}{N} = \frac{3}{2} \left(\frac{R}{N} \right) T = \frac{3}{2} kT$$

where $k = \frac{R}{N}$ is Boltzmann constant. Thus energy $\propto T$.

34. Write the formula for the pressure of an ideal gas in terms of molecular mass, number of molecules and their velocity on the basis of kinetic theory and with the help of it, establish the relation between kinetic energy of the molecules and temperature of the gas.

Sol. The pressure exerted by an ideal gas is given by

$$P = \frac{1}{3} nm \overline{v^2}$$

Multiplying both sides by V

$$PV = \frac{1}{3} nmV \overline{v^2}$$

$$PV = \frac{1}{3} N m \overline{v^2} \quad \dots(i)$$

where $N = (nV)$ is the number of molecules in the sample.

The internal energy E of an ideal gas is purely kinetic

$$E = N \frac{1}{2} m v^2 \quad \dots(\text{ii})$$

Using equation (ii) in (i), we get

$$PV = \left(\frac{2}{3}\right) E \quad \dots(\text{iii})$$

Comparing equation (iii) with the ideal gas equation, we get

$$\frac{2}{3} E = N k_B T$$

$$E = \frac{3}{2} N k_B T$$

or $\frac{E}{N} = \frac{1}{2} m \overline{v^2} = \frac{3}{2} k_B T \quad \dots(\text{iv})$

35. Two vessels A and B are identical. A has 1 gm hydrogen at 0°C and B has 1 gm nitrogen at 0°C .

- (i) Which vessel does contain more molecules and how much?
- (ii) In which vessel is the pressure of the gas higher and how much?
- (iii) In which vessel is the average speed of molecules larger and how much?

Sol. (i) Vessel A contains 14 times more (hydrogen) molecules than the nitrogen molecules contained in vessel B , because the number of molecules in 1 gm of hydrogen is 14 times the number of molecules in 1 gm oxygen.

- (ii) At a given temperature, the average kinetic energy per molecule $\left(\frac{3}{2} k_B T\right)$ is independent of the nature of the gas. Now, from the ideal gas equation $PV = n k_B T$ for n molecules, the pressure of the gas is

$$P = \frac{n k_B T}{V}$$

Thus, for a given volume, the gas pressure is proportional to the number of molecules. Hence, pressure in A will be 14 times higher than that in B .

- (iii) At a given temperature $v_{\text{rms}} \propto \frac{1}{\sqrt{M}}$, where M is molecular mass of the gas. Since, the molecular mass of hydrogen is $\left(\frac{1}{14}\right)$ th the molecular mass of oxygen, the rms speed of hydrogen in vessel A is $\sqrt{14}$ times larger than that of nitrogen in vessel B .

36. A vessel contains two non-reacting gases, neon (monatomic) and oxygen (diatomic). The ratio of their partial pressures is 3 : 2. Find the ratio of (a) number of molecules, and (b) mass density of Ne and O_2 in the vessel. The atomic mass of Ne is 20.2 and the molecular mass of O_2 is 32.0.

Sol. The partial pressure of a gas in a mixture of gases filled in a vessel is that pressure which the gas would have if it alone occupied the whole vessel at the same temperature. (The total pressure of a mixture of non-reacting gases is the sum of partial pressures of its constituent gases).

Since, V and T are common for the gases neon and oxygen (assumed ideal), we can write

$$P_1 V = \mu_1 R T \text{ and } P_2 V = \mu_2 R T$$

where P_1 and P_2 are the partial pressure, μ_1 and μ_2 the number of moles of neon and oxygen respectively in the vessel. Thus,

$$\frac{P_1}{P_2} = \frac{\mu_1}{\mu_2} \text{ as } \frac{P_1}{P_2} = \frac{3}{2} \text{ (given)}$$

$$\therefore \frac{\mu_1}{\mu_2} = \frac{3}{2}$$

(a) If n_1 and n_2 be the number of molecules of neon and oxygen respectively, and N be the Avogadro number, then by definition, we have

$$\mu_1 = \frac{n_1}{N} \text{ and } \mu_2 = \frac{n_2}{N}$$

$$\therefore \frac{n_1}{n_2} = \frac{\mu_1}{\mu_2} = \frac{3}{2}$$

(b) If m_1 and m_2 be the masses, and M_1 and M_2 the molecular masses of neon and oxygen respectively, then we have

$$\mu_1 = \frac{m_1}{M_1} \text{ and } \mu_2 = \frac{m_2}{M_2}$$

$$\frac{\rho_1}{\rho_2} = \frac{m_1/v}{m_2/v} = \left(\frac{\mu_1}{\mu_2}\right) \left(\frac{M_1}{M_2}\right) = \frac{3}{2} \times \frac{20.2}{32.0} = 0.947$$

37. An oxygen cylinder of volume 30 litres has an initial gauge pressure of 15 atm and a temperature of 27°C. After some oxygen is withdrawn from the cylinder, the gauge pressure drops to 11 atm and its temperature drops to 17°C. Estimate the mass of oxygen taken out of the cylinder.

($R = 8.3 \text{ J mol}^{-1} \text{ K}^{-1}$; molecular mass M of $\text{O}_2 = 32$)

Sol. Let P_1 and T_1 be the initial pressure and absolute temperature of oxygen in the cylinder, and P_2 and T_2 their values after some oxygen is withdrawn. Let μ_1 the number of moles of oxygen initially present in the cylinder and μ_2 the number of moles left over. The volume V (say) remains unchanged. Now, writing gas equation, we have

$$P_1 V = \mu_1 R T_1$$

$$\text{and } P_2 V = \mu_2 R T_2$$

Here, $P_1 = 15 \text{ atm} = 15 \times 1.01 \times 10^5 \text{ Nm}^{-2}$, $P_2 = 11 \times 1.01 \times 10^5 \text{ Nm}^{-2}$, $T_1 = 27 + 273 = 300 \text{ K}$,

$T_2 = 17 + 273 = 290 \text{ K}$, $V = 30 \text{ litre} = 30 \times 10^{-3} \text{ m}^3$ and $R = 8.3 \text{ J mol}^{-1} \text{ K}^{-1}$. Therefore,

$$\mu_1 = \frac{P_1 V}{R T_1} = \frac{(15 \times 1.01 \times 10^5) \times (30 \times 10^{-3})}{8.3 \times 300} = 18.25$$

$$\text{and } \mu_2 = \frac{P_2 V}{R T_2} = \frac{(11 \times 1.01 \times 10^5) \times (30 \times 10^{-3})}{8.3 \times 290} = 13.84$$

\therefore Number of moles of oxygen taken out of the cylinder is

$$\mu_1 - \mu_2 = 18.25 - 13.84 = 4.41$$

The molecular mass of O_2 is 32 g.

Hence, the mass of oxygen taken out of the cylinder is

$$m = (\mu_1 - \mu_2)M = 4.41 \times 32 \text{ g} = 141.12 \text{ g}$$

38. A vessel A contains hydrogen and another vessel B whose volume is twice of A contains same mass of oxygen at the same temperature. Compare

- (i) Average translational kinetic energies of hydrogen and oxygen molecules,
- (ii) Root-mean square speeds of the molecules,
- (iii) Pressures of gases in A and B .

(Molecular weights of hydrogen and oxygen are 2 and 32 respectively).

Sol. (i) For all gases at the same temperature, the kinetic energy per molecule is the same $\left(\frac{3}{2}k_B T\right)$. Since, gases in both vessels are at the same temperature, the average kinetic energy per molecule is the same (1 : 1).

(ii) We know that $v_{\text{rms}} = \sqrt{\frac{3RT}{M}}$

$$\therefore \frac{(v_{\text{rms}})_H}{(v_{\text{rms}})_O} = \sqrt{\frac{M_O}{M_H}} = \sqrt{\frac{32}{2}} = 4 : 1$$

(iii) According to the kinetic theory, we have

$$P = \frac{1}{3} \frac{mn}{V} \overline{v^2} = \frac{1}{3} \frac{M}{V} \overline{v^2}$$

Masses of both the gases hydrogen and oxygen are equal.

Hence, the ratio of pressures is

$$\frac{P_H}{P_O} = \frac{\overline{v_H^2}}{\overline{v_O^2}} \times \frac{V_O}{V_H} = \frac{(v_{\text{rms}}^2)_H}{(v_{\text{rms}}^2)_O} \times \frac{V_O}{V_H} = \frac{16}{1} \times \frac{2}{1} = 32 : 1$$

39. What do you mean by the degree of freedom and law of equipartition of energy?

Sol. Degree of freedom of a system refers to the possible independent motions a system can have or number of possible independent ways in which a system can have energy.

- (i) Monoatomic molecule has 3 degrees of freedom all translational.
- (ii) A diatomic molecule has 5 degrees of freedom 3 translational and 2 rotational.
- (iii) A diatomic molecule that is free to vibrate will have 7 degrees of freedom (3 + 2 + 2).
- (iv) A nonlinear polyatomic molecule has 6 degrees of freedom 3 translational and 3 rotational.

The law of equipartition of energy states that if a system is in equilibrium at absolute temperature T , the total energy is distributed equally in different energy modes of absorption. The energy in each mode being

equal to $\frac{1}{2}k_B T$.

Each translation and rotational degree of freedom corresponds to one energy mode of absorption and has energy $\frac{1}{2}k_B T$. Each vibrational frequency has two modes of energy with corresponding energy equal to

$$2 \times \frac{1}{2}k_B T = k_B T.$$

40. Explain the concept of mean free path.

Sol. The size of gas molecules is very small and they are undergoing collisions, due to which their motion is not straight or we can say that the path of molecules keep on getting deflected due to collisions. Suppose the molecules of a gas are spheres of diameter d .

Let us focus on a single molecule having average speed $\langle v \rangle$. It will suffer collision with any molecule that comes with in a distance d , (between the centres).

In Δt time the volume sweeps by the molecule is given by

Volume = Area \times distance travelled by the molecule in Δt time

$$V = \pi d^2 \langle v \rangle \Delta t \quad \dots(i)$$

Equation (i) gives the volume in which any other molecule can collide with it. If n is number of molecules per unit volume, then the molecule will suffer $n\pi d^2 \langle v \rangle \Delta t$ collisions in Δt time. The rate of collision is

$$\frac{n\pi d^2 \langle v \rangle \Delta t}{\Delta t} = n\pi d^2 \langle v \rangle$$

and the time (τ) between two successive collision is given by

$$\tau = \frac{1}{(n\pi \langle v \rangle d^2)} \quad \dots(ii)$$

The average distance between two successive collisions, called the mean free path (l) which is given by

$$l = \langle v \rangle \tau = \frac{1}{(n\pi d^2)} \quad \dots(iii)$$

While doing the above derivation we have not considered the motion of other molecules. We assumed them to be at rest. But actually all are moving. Hence $\langle v \rangle$ is replaced by $\langle v_r \rangle$ i.e., average relative velocity of the molecules. Hence more exact treatment gives

$$l = \frac{1}{\sqrt{2}n\pi d^2} \quad \dots(iv)$$

The mean free path of equation (iv) gives its dependence on the number density and size of molecules.

41. On the basis of law of equipartition of energy, calculate the molar specific heats of monoatomic and diatomic gases at constant volume and pressure separately.

Sol. A monoatomic gas has only three translational degrees of freedom therefore, its average energy at temperature T is given by

$$E = \frac{3}{2} k_B T$$

Total internal energy of one mole of such gas is

$$U = \frac{3}{2} k_B T \times N_A \quad \dots(i)$$

where N_A is Avogadro's number.

As we know, $k_B = \frac{R}{N_A}$, substituting in equation (i), we get

$$U = \frac{3}{2}RT$$

$$\frac{dU}{dT} = \frac{3}{2}R$$

The molar specific heat at constant volume, C_v is

$$\frac{dU}{dT} = C_v \text{ (monoatomic gas)} = \frac{3}{2}R$$

For an ideal gas

$$C_p - C_v = R$$

where C_p is the molar specific heat at constant pressure. Thus,

$$C_p = R + C_v = R + \frac{3}{2}R = \frac{5}{2}R$$

Ratio of specific heats $\gamma = \frac{C_p}{C_v} = \frac{5}{3}$

A diatomic molecule is treated as a rigid rotator with 5 degrees of freedom, 3 translational and 2 rotational. So the average energy of a molecule of diatomic gas at temperature T is given by

$$E = \frac{5}{2}k_B T$$

and the total internal energy of a mole of diatomic gas is

$$U = \frac{5}{2}k_B T \times N_A = \frac{5}{2}RT$$

and hence, molar specific heat C_v (at constant volume) and molar specific heat C_p (at constant pressure) is given by

$$C_v = \frac{dU}{dT} = \frac{5}{2}R$$

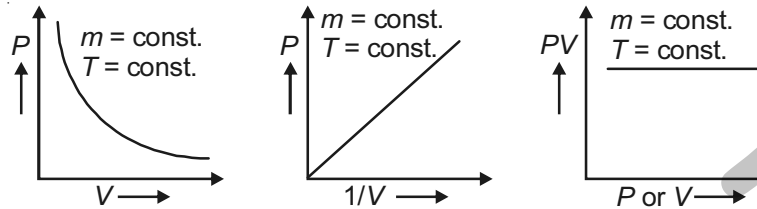
$$C_v = \frac{5}{2}R$$

$$C_p = R + C_v = R + \frac{5}{2}R = \frac{7}{2}R$$

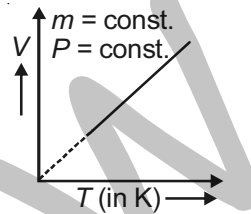
$$\gamma = \frac{C_p}{C_v} = \frac{7}{5}$$

42. Explain Boyle's law and Charles's law and draw the corresponding graphs related to P , V and T .

Sol. Boyle's law : According to it, for a given mass of an ideal gas at constant temperature, the volume of a gas is inversely proportional to its pressure, i.e., $V \propto \frac{1}{P}$ if mass of gas and $T = \text{constant}$



Charles's law : According to it, for a given mass of an ideal gas at constant pressure, volume of a gas is directly proportional to its absolute temperature i.e., $V \propto T$, if m and P are constant.



43. On the basis of kinetic theory of gases, explain the kinetic interpretation of temperature.

Sol. The pressure exerted by an ideal gas is given by

$$P = \frac{1}{3}nm\overline{v^2}$$

Multiplying both sides by V

$$PV = \frac{1}{3}nmV\overline{v^2}$$

$$PV = \frac{1}{3}Nmv^2 \quad \dots(i)$$

where $N = (nV)$ is the number of molecules in the sample.

The internal energy E of an ideal gas is purely kinetic

$$E = N \frac{1}{2}mv^2 \quad \dots(ii)$$

Using equation (ii) in (i), we get

$$PV = \left(\frac{2}{3}\right)E \quad \dots(iii)$$

Comparing equation (iii) with the ideal gas equation, we get

$$\frac{2}{3}E = Nk_B T$$

$$E = \frac{3}{2} Nk_B T$$

$$\text{or } \frac{E}{N} = \frac{1}{2} m \overline{v^2} = \frac{3}{2} k_B T \quad \dots(\text{iv})$$

From equation (iv) we can see that the average kinetic energy of a molecule is proportional to the absolute temperature of the gas. It is independent of pressure, volume or the nature of the ideal gas. This is a fundamental result relating the temperature to the internal energy of a molecule. This is kinetic interpretation of temperature.

The *rms* velocity (root mean square velocity) is defined as the square root of the mean of the squares of the random velocities of the individual molecules of a gas

$$v_{\text{rms}} = \sqrt{\overline{v^2}}$$

As from the above equation we can see that $\overline{v^2} \propto T$

$$\therefore v_{\text{rms}} \propto \sqrt{T}$$

44. Establish a relation between kinetic energy of the molecules of a gas and temperature and show that the root mean square velocity of the molecules of a gas is directly proportional to the square root of the absolute temperature of the gas.

Sol. The pressure exerted by an ideal gas is given by

$$P = \frac{1}{3} n m \overline{v^2}$$

Multiplying both sides by V

$$PV = \frac{1}{3} n m V \overline{v^2}$$

$$PV = \frac{1}{3} N m \overline{v^2} \quad \dots(\text{i})$$

where $N = (nV)$ is the number of molecules in the sample.

The internal energy E of an ideal gas is purely kinetic

$$E = N \frac{1}{2} m \overline{v^2} \quad \dots(\text{ii})$$

Using equation (ii) in (i), we get

$$PV = \left(\frac{2}{3}\right) E \quad \dots(\text{iii})$$

Comparing equation (iii) with the ideal gas equation, we get

$$\frac{2}{3} E = Nk_B T$$

$$E = \frac{3}{2} Nk_B T$$

$$\text{or } \frac{E}{N} = \frac{1}{2} m \overline{v^2} = \frac{3}{2} k_B T \quad \dots(\text{iv})$$

The rms velocity (root mean square velocity) is defined as the square root of the mean of the squares of the random velocities of the individual molecules of a gas

$$v_{\text{rms}} = \sqrt{\overline{v^2}}$$

As from the above equation we can see that $\overline{v^2} \propto T$

$$\therefore v_{\text{rms}} \propto \sqrt{T}$$

SARASWATI

Multiple Choice Questions

- If pressure of a gas contained in a closed vessel is increased by 0.4% when heated by 1°C , the initial temperature must be
 (a) 250 K (b) 250°C
 (c) 2500 K (d) 25°C
- The pressure P , volume V and temperature T of a gas in the jar A and the other in the jar B is at pressure $2P$, volume $V/4$ and temperature $2T$, then the ratio of the number of molecules in the jar A and B will be
 (a) 1:1 (b) 1:2
 (c) 2:1 (d) 4:1
- Air is filled in a bottle at atmospheric pressure and it is corked at 35°C . If the cork can come out at 3 atmospheric pressure then up to what temperature should the bottle be heated in order to remove the cork
 (a) 325.5°C (b) 851°C
 (c) 651°C (d) None of these
- The pressure of an ideal gas is written as $P = \frac{2E}{3V}$. Here E refers to
 (a) translational kinetic energy
 (b) rotational kinetic energy
 (c) vibrational kinetic energy
 (d) total kinetic energy
- Three containers of the same volume contain three different gases. The masses of the molecules are m_1 , m_2 and m_3 and the number of molecules in their respective containers are N_1 , N_2 and N_3 . The gas pressure in the containers are P_1 , P_2 and P_3 respectively. All the gases are now mixed and put in one of the containers. the pressure P of mixture will be
 (a) $P < (P_1 + P_2 + P_3)$
 (b) $P = \frac{P_1 + P_2 + P_3}{3}$
 (c) $P = P_1 + P_2 + P_3$
 (d) $P > (P_1 + P_2 + P_3)$
- A cubical box with porous walls containing an equal number of O_2 and H_2 molecules is placed in a larger evacuated chamber. The entire system is maintained at constant temperature T . The ratio of v_{rms} of O_2 molecules to that of the v_{rms} of H_2 molecules, found in the chamber outside the box after a short interval is
 (a) $\frac{1}{2\sqrt{2}}$ (b) $\frac{1}{4}$
 (c) $\frac{1}{\sqrt{2}}$ (d) $\sqrt{2}$
- Three closed vessels A , B and C are at the same temperature T and contain gases which obey the Maxwellian distribution of velocities. Vessel A contains only O_2 , B only N_2 and C a mixture of equal quantities of O_2 and N_2 . If the average speed of the O_2 molecules in vessel A is V_1 , that of the N_2 molecules in vessel B is V_2 , the average speed of the O_2 molecules in vessel C is
 (a) $\frac{(V_1 + V_2)}{2}$ (b) V_1
 (c) $(V_1 v_2)^{1/2}$ (d) $\sqrt{\frac{3kT}{M}}$
- The temperature of argon, kept in a vessel, is raised by 1°C at a constant volume. the total heat supplied to the gas is a combination of translational and rotational energies. Their respective shares are
 (a) 60% and 40% (b) 40% and 60%
 (c) 50% and 50% (d) 100% and 0%
- Two containers of equal volume contain the same gas at pressure P_1 and P_2 and absolute temperatures T_1 and T_2 respectively. On joining the vessels, the gas reaches a common pressure p and a common temperature T . The ratio p/T is equal to :
 (a) $\frac{p_1}{T_1} + \frac{p_2}{T_2}$ (b) $\frac{1}{2} \left[\frac{p_1}{T_1} + \frac{p_2}{T_2} \right]$
 (c) $\frac{p_1 T_2 + p_2 T_1}{T_1 + T_2}$ (d) $\frac{p_1 T_2 - p_2 T_1}{T_1 - T_2}$
- An air bubble doubles in radius on rising from the bottom of a lake to its surface. If the atmospheric pressure is equal to that of a column of water of height H , the depth of the lake is
 (a) H (b) $2H$
 (c) $7H$ (d) $8H$

11. If pressure of CO_2 (real gas) in a container is given by $P = \frac{RT}{2V-b} - \frac{a}{4b^2}$, then mass of the gas in container is
 (a) 11 g (b) 22 g
 (c) 33 g (d) 44 g
12. The speed of sound in oxygen (O_2) at a certain temperature is 460 ms^{-1} . The speed of sound in helium (He) at the same temperature will be (assume both gases to be ideal)
 (a) 1421 ms^{-1} (b) 500 ms^{-1}
 (c) 650 ms^{-1} (d) 330 ms^{-1}
13. A mixture of 2 moles of helium gas (atomic mass = 4 amu) and 1 mole of argon gas (atomic mass = 40 amu) is kept at 300 K in a container. The ratio of the rms speeds is $\left(\frac{v_{rms}(\text{helium})}{v_{rms}(\text{argon})}\right)$
 (a) 0.32 (b) 0.45
 (c) 2.24 (d) 3.16
14. Two non-reactive monoatomic ideal gases have their atomic masses in the ratio 2:3. The ratio of their partial pressures, when enclosed in a vessel kept at a constant temperature, is 4:3. The ratio of their densities is :
 (a) 1:4 (b) 1:2
 (c) 6:9 (d) 8:9
15. A perfect gas at 27°C is heated at constant pressure so as to double its volume. The final temperature of gas will be close to
 (a) 327°C (b) 200°C
 (c) 54°C (d) 300°C
16. At room temperature a diatomic gas is found to have an r.m.s. speed of 1930 ms^{-1} . The gas is :
 (a) H_2 (b) Cl_2
 (c) O_2 (d) F_2
17. Using equipartition of energy, the specific heat (in $\text{J kg}^{-1} \text{K}^{-1}$) of aluminium at room temperature can be estimated to be (atomic weight of aluminium = 27)
 (a) 410 (b) 25
 (c) 1850 (d) 925
18. A diatomic gas is heated at constant pressure. The fraction of the heat energy is used to increase the internal energy is
 (a) $\frac{3}{5}$ (b) $\frac{3}{7}$
 (c) $\frac{5}{7}$ (d) $\frac{5}{9}$
19. For a gas if ratio of specific heat at constant pressure and volume is γ then the value of degree of freedom is
 (a) $\frac{3\gamma-1}{2\gamma-1}$ (b) $\frac{2}{\gamma-1}$
 (c) $\frac{9}{2}(\gamma-1)$ (d) $\frac{25}{2}(\gamma-1)$
20. Two ideal gas at absolute temperature T_1 and T_2 are mixed. There is no loss of energy. The masses of the molecules are m_1 and m_2 and the number of molecules in the gases are n_1 and n_2 respectively. The temperature of the mixture is
 (a) $\frac{T_1+T_2}{2}$ (b) $\frac{T_1+T_2}{n_1n_2}$
 (c) T_1+T_2 (d) $\frac{n_1T_1+n_2T_2}{n_1+n_2}$
21. Two thermally insulated vessels 1 and 2 are filled with air at temperatures (T_1, T_2), volume (V_1, V_2) and pressure (P_1, P_2) respectively. If the valve joining the two vessels is opened the temperature inside the vessel at equilibrium will be :
 (a) T_1+T_2 (b) $\frac{(T_1+T_2)}{2}$
 (c) $\frac{T_1T_2(P_1V_1+P_2V_2)}{P_1V_1T_2+P_2V_2T_1}$ (d) $\frac{T_1T_2(P_1V_1+P_2V_2)}{P_1V_1T_1+P_2V_2T_2}$
22. A gas mixture consists of 2 moles of oxygen and 4 moles of argon at temperature T. Neglecting all vibrational modes, the total internal energy of the system is :
 (a) 4 RT (b) 15 RT
 (c) 9 RT (d) 11 RT
23. A vessel containing one gram mole of oxygen at 27°C and 1 atm pressure is thermally insulated and moles with a constant speed v_0 . It is then suddenly stopped and this results in a rise of temperature of the gas by 1°C the speed v_0 is :
 (a) 63.03 m/s (b) 30.63 m/s
 (c) 36.03 m/s (d) 33.06 m/s
24. A gaseous mixture consists of 16 g of helium and 16 g of oxygen. The ratio $\frac{C_p}{C_v}$ of the mixture is :
 (a) 1.62 (b) 1.59
 (c) 1.54 (d) 1.4
25. If C_p and C_v denote the specific heats of

nitrogen per unit mass at constant pressure and constant volume respectively, then

- (a) $C_p - C_v = 28R$
 (b) $C_p - C_v = R/28$
 (c) $C_p - C_v = R/14$
 (d) $C_p - C_v = R$

26. Consider an ideal gas confined in an isolated closed chamber. As the gas undergoes an adiabatic expansion, the average time of collision between molecules increases as V^q , where V is the volume of the gas. The value of q

is $\left(\gamma = \frac{C_p}{C_v}\right)$

- (a) $\frac{\gamma+1}{2} s$ (b) $\frac{\gamma-1}{2}$
 (c) $\frac{3\gamma+5}{6}$ (d) $\frac{3\gamma-5}{6}$

27. A vessel has 6g of hydrogen at pressure P and temperature 500 K. A small hole is made in it so that hydrogen leaks out. How much hydrogen leaks out if the final pressure is $\frac{P}{2}$ and temperature falls to 300 K?

- (a) 2g (b) 3g
 (c) 4g (d) 1g

28. A cylinder contains 12 litres of oxygen at 20°C and 15 atm pressure. The temperature of the gas is raised to 35°C and its volume increased to 17 litres. What is the final pressure of gas (in atm)?

- (a) 9 (b) 11
 (c) 15 (d) 17

29. The temperature of the gas contained in a closed vessel increases by 1°C when pressure of the gas is increased by 1%. The initial temperature of the gas is

- (a) 100 K (b) 100°C
 (c) 200 K (d) 250°C

30. A gas at absolute temperature 300 K has pressure $4 \times 10^{-10} \text{ N/m}^2$. The number of molecules per cm^3 is of the order of (Boltzmann constant, $k_B = 1.38 \times 10^{-23} \text{ J/K}$)

- (a) 100 (b) 10^5
 (c) 10^8 (d) 10^{11}

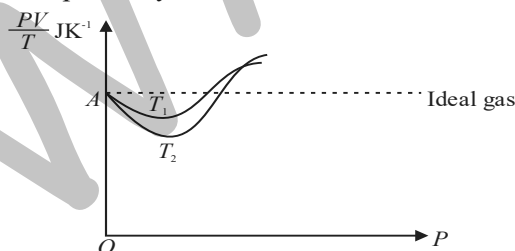
31. Two containers of equal volume contain the same gas at pressures P_1 and P_2 and absolute temperatures T_1 and T_2 respectively. On joining the vessels, the gas reaches a common pressure P and a common temperature T. The ratio $\frac{P}{T}$ is

- (a) $\frac{P_1}{T_1} + \frac{P_2}{T_2}$ (b) $\frac{1}{2} \left[\frac{P_1}{T_1} + \frac{P_2}{T_2} \right]$
 (c) $\frac{P_1 T_2 + P_2 T_1}{T_1 + T_2}$ (d) $\frac{P_1 T_2 - P_2 T_1}{T_1 - T_2}$

32. A balloon contains 1500 m^3 of helium at 27°C and 4 atmospheric pressure. The volume of helium at -3°C temperature and 2 atmospheric pressure will be

- (a) 1500 m^3 (b) 1700 m^3
 (c) 1900 m^3 (d) 2700 m^3

33. Given is the graph between $\frac{PV}{T}$ and P for 1g of oxygen gas at two different temperatures T_1 and T_2 , as shown in figure. Given, density of oxygen = 1.427 kg m^{-3} . The value of PV/T at the point A and the relation between T_1 and T_2 are respectively



- (a) 0.259 J K^{-1} and $T_1 < T_2$
 (b) 8.314 $\text{g J mol}^{-1} \text{K}^{-1}$ and $T_1 > T_2$
 (c) 0.259 g J K^{-1} and $T_1 > T_2$
 (d) 4.28 g J K^{-1} and $T_1 < T_2$

34. Two perfect gases having masses m_1 and m_2 at temperatures T_1 and T_2 respectively are mixed without any loss of energy. If the molecular weights of the gases are M_1 and M_2 respectively, then the final temperature of the mixture is

- (a) $\frac{(m_1 T_1 + m_2 T_2)}{m_1 + m_2}$
 (b) $\frac{(M_1 T_1 + M_2 T_2)}{M_1 + M_2}$
 (c) $\frac{\left[\left(\frac{m_1 T_1}{M_1} \right) + \left(\frac{m_2 T_2}{M_2} \right) \right]}{\left[\left(\frac{m_1}{M_1} \right) + \left(\frac{m_2}{M_2} \right) \right]}$
 (d) $\frac{\left[\frac{M_1 T_1}{m_1} + \frac{M_2 T_2}{m_2} \right]}{\left[\left(\frac{M_1}{m_1} \right) + \left(\frac{M_2}{m_2} \right) \right]}$

35. The kinetic theory of gases gives the formula $PV = \frac{1}{3} Nmv^2$ for the pressure P exerted by a gas enclosed in a volume V. The term Nm represents
- the mass of a mole of the gas
 - the mass of the gas present in the volume V
 - the average mass of one molecule of the gas
 - the total number of molecules present in volume V
36. Which one of the following is not an assumption of kinetic theory of gases?
- The volume occupied by the molecules of the gas is negligible
 - The force of attraction between the molecules is negligible
 - The collision between the molecules are elastic.
 - All molecules have same speed.
37. The average kinetic energy of the molecules of a low density gas at 27°C is (Boltzmann constant $= 1.38 \times 10^{-23} \text{ J K}^{-1}$)
- $3.1 \times 10^{-20} \text{ J}$
 - $3.5 \times 10^{-21} \text{ J}$
 - $5.3 \times 10^{-18} \text{ J}$
 - $6.21 \times 10^{-21} \text{ J}$
38. At 27°C temperature, the kinetic energy of an ideal gas E_1 . If the temperature is increased to 327°C , then kinetic energy would be
- $\frac{E_1}{2}$
 - $\frac{E_1}{\sqrt{2}}$
 - $\sqrt{2}E_1$
 - $2E_1$
39. If f is the degree of freedom, then C_p / C_v is
- $1 - \frac{2}{f}$
 - $1 + \frac{2}{f}$
 - $1 + \frac{f}{2}$
 - $1 - \frac{f}{2}$
40. For nitrogen, $C_p - C_v = x$ and for argon, $C_p - C_v = y$. The relation between x and y is given as
- $x = y$
 - $x = 7y$
 - $y = 7x$
 - $x = (1/2)y$
41. A gaseous mixture consists of 16 g of helium and 16 g of oxygen. The ratio C_p / C_v of the mixture is
- 1.4
 - 1.54
 - 1.59
 - 1.62
42. One keg of a diatomic gas is at a pressure of $8 \times 10^4 \text{ N m}^{-2}$. The density of the gas is 4 kg m^{-3} . What is the energy of the gas due to its thermal motion?
- $3 \times 10^4 \text{ J}$
 - $5 \times 10^4 \text{ J}$
 - $6 \times 10^4 \text{ J}$
 - $7 \times 10^4 \text{ J}$
43. The heat capacity per mole of water is (R is universal gas constant)
- 9R
 - $\frac{9}{2}R$
 - 6R
 - 5R
44. If for a gas $\frac{R}{C_v} = 0.67$, this gas is made up of molecules which are
- monatomic
 - diatomic
 - polyatomic
 - mixture of diatomic and polyatomic molecules
45. Mean free path of a gas molecule is
- inversely proportional to number of molecules per unit volume
 - inversely proportional to diameter of the molecule
 - directly proportional to the square root of the absolute temperature
 - directly proportional to the pressure

Answer Key

- | | | |
|-------|-------|-------|
| 1. A | 2. D | 3. C |
| 4. A | 5. C | 6. B |
| 7. B | 8. D | 9. B |
| 10. C | 11. C | 12. A |
| 13. D | 14. D | 15. A |
| 16. A | 17. D | 18. C |
| 19. B | 20. D | 21. C |
| 22. D | 23. C | 24. A |
| 25. B | 26. A | 27. D |
| 28. B | 29. A | 30. B |
| 31. B | 32. D | 33. C |
| 34. C | 35. B | 36. D |
| 37. D | 38. D | 39. B |
| 40. A | 41. D | 42. B |
| 43. A | 44. A | 45. A |

JEE & NEET Previous Year(Kinetic Theory & Oscillation)

NEET & AIPMT

1. Average velocity of a particle executing SHM in one complete vibration is: [NEET 2019]

(a) $\frac{A\omega^2}{2}$ (b) Zero
(c) $\frac{A\omega}{2}$ (d) $A\omega$

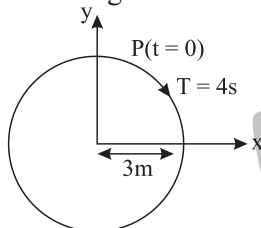
2. The displacement of a particle executing simple harmonic motion is given by

$$y = A_0 + A \sin \omega t + B \cos \omega t$$

Then the amplitude of its oscillation is given by: [NEET 2019]

(a) $\sqrt{A_0^2 + (A+B)^2}$ (b) $A+B$
(c) $A_0 + \sqrt{A^2 + B^2}$ (d) $\sqrt{A^2 + B^2}$

3. The radius of circle, the period of revolution, initial position and sense of revolution are indicated in the fig. [NEET 2019]



y - projection of the radius vector of rotating particle P is :

(a) $y(t) = 3 \cos\left(\frac{3\pi t}{2}\right)$, where y in m
(b) $y(t) = 3 \cos\left(\frac{\pi t}{2}\right)$, where y in m
(c) $y(t) = -3 \cos 2\pi t$, where y in m
(d) $y(t) = 4 \sin\left(\frac{\pi t}{2}\right)$, where y in m

4. A pendulum is hung from the roof of a sufficiently high building and is moving freely to and fro like a simple harmonic oscillator. The acceleration of the bob of the pendulum is 20 m s^{-2} at a distance of 5 m from the mean position. The time period of oscillation is [NEET 2018]

(a) 2 s (b) π s
(c) 2π s (d) 1 s

5. A particle executes linear simple harmonic motion with an amplitude of 3 cm. When the particle is at 2 cm from the mean position, the

magnitude of its velocity is equal to that of its acceleration. Then, its time period in seconds is

[NEET 2017]

(a) $\frac{\sqrt{5}}{\pi}$ (b) $\frac{\sqrt{5}}{2\pi}$
(c) $\frac{4\pi}{\sqrt{5}}$ (d) $\frac{2\pi}{\sqrt{3}}$

6. A body of mass m is attached to the lower end of a spring whose upper end is fixed. The spring has negligible mass. When the mass m is slightly pulled down and released, it oscillates with a time period of 3 s. When the mass m is increased by 1 kg, the time period of oscillations becomes 5 s. The value of m in kg is

[NEET 2016]

(a) $\frac{3}{4}$ (b) $\frac{4}{3}$
(c) $\frac{16}{9}$ (d) $\frac{9}{16}$

7. When two displacements represented by $y_1 = a \sin(\omega t)$ and $y_2 = b \cos(\omega t)$ are superimposed, the motion is [AIPMT 2015]

(a) not a simple harmonic
(b) simple harmonic with amplitude $\frac{a}{b}$
(c) simple harmonic with amplitude $\sqrt{a^2 + b^2}$
(d) simple harmonic with amplitude $\frac{(a+b)}{2}$

8. A particle is executing SHM along a straight line. Its velocities at distances X_1 and X_2 from the mean position are V_1 and V_2 , respectively. Its time period is [AIPMT 2015]

(a) $2\pi \sqrt{\frac{X_1^2 + X_2^2}{V_1^2 + V_2^2}}$ (b) $2\pi \sqrt{\frac{X_2^2 - X_1^2}{V_1^2 - V_2^2}}$
(c) $2\pi \sqrt{\frac{X_1^2 - X_2^2}{V_1^2 - V_2^2}}$ (d) $2\pi \sqrt{\frac{V_1^2 - V_2^2}{X_1^2 - X_2^2}}$

9. A particle is executing a simple harmonic motion. Its maximum acceleration is α and maximum velocity is β . Then, its time period of vibration will be [AIPMT 2015]

(a) $\frac{\beta^2}{\alpha^2}$ (b) $\frac{\alpha}{\beta}$
(c) $\frac{\beta^2}{\alpha}$ (d) $\frac{2\pi\beta}{\alpha}$

10. An air column, closed at one end and open at the other, resonates with a tuning fork when the smallest length of the column is 50 cm. The next larger length of the column resonating with the same tuning fork is [AIPMT 2015]

(a) 100 cm (b) 150 cm
(c) 200 cm (d) 66.7 cm

11. A string is stretched between fixed points separated by 75.0 cm. It is observed to have resonant frequencies of 420 Hz and 315 Hz. There are no other resonant frequencies between these two. The lowest resonant frequency for this strings is [AIPMT 2015]

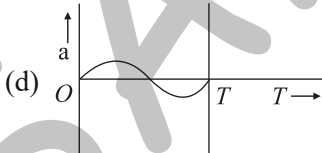
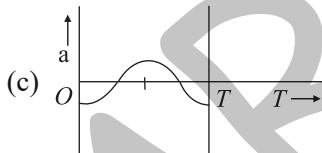
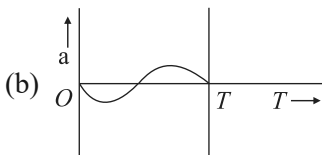
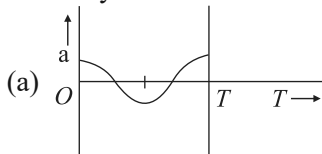
(a) 155 Hz (b) 205 Hz
(c) 10.5 Hz (d) 105 Hz

12. The oscillation of a body on a smooth horizontal surface is represented by the equation,
 $X = A \cos(\omega t)$

where, X = displacement at time t

ω = frequency of oscillation

Which one of the following graphs shows correctly the variation a with t ? [AIPMT 2014]



Here, a = acceleration at time t
 T = Time period

13. The damping force on an oscillator is directly proportional to the velocity. The units of the constant of proportionality are [AIPMT 2012]

(a) kg ms^{-1} (b) kg ms^{-2}
(c) kg s^{-1} (d) kg s

14. Out of the following functions representing motion of a particle which represents SHM? [AIPMT 2011]

I. $y = \sin \omega t - \cos \omega t$

II. $y = \sin^3 \omega t$

III. $y = 5 \cos\left(\frac{3\pi}{4} - 3\omega t\right)$

IV. $y = 1 + \omega t + \omega^2 t^2$

(a) Only (IV) does not represent SHM

(b) (I) and (III)

(c) (I) and (II)

(d) Only (I)

15. The displacement of a particle along the x-axis is given by $x = a \sin^2 \omega t$. The motion of the particle corresponds to [AIPMT 2010]

(a) simple harmonic motion of frequency ω/p

(b) simple harmonic motion of frequency $3\omega/2p$

(c) non-simple harmonic motion

(d) simple harmonic motion of frequency to $\omega/2p$

16. The period of oscillation of a mass M suspended from a spring of negligible mass is T . If along with it another mass M is also suspended, the period of oscillation will now be [AIPMT 2010]

(a) T (b) $T/\sqrt{2}$

(c) $2T$ (d) $\sqrt{2} T$

17. Which one of the following equations of motion represents simple harmonic motion? [AIPMT 2009]

(a) Acceleration = $-k_0 x + k_1 x^2$

(b) Acceleration = $-k(x + a)$

(c) Acceleration = $k(x + a)$

(d) Acceleration = kx

(where, k, k_0, k_1 and a are all positive.)

18. A simple pendulum performs simple harmonic motion about $x = 0$ with an amplitude a and time period T . The speed of the pendulum at $x = \frac{a}{2}$ will be [AIPMT 2009]

(a) $\frac{\pi a \sqrt{3}}{2T}$ (b) $\frac{\pi a}{T}$

(c) $\frac{3\pi^2 a}{T}$ (d) $\frac{\pi a \sqrt{3}}{T}$

19. Two simple harmonic motions of angular frequency 100 rad s^{-1} and 1000 rad s^{-1} have the same displacement amplitude. The ratio of their maximum accelerations is [AIPMT 2008]

(a) 1 : 10 (b) 1 : 10^2

(c) 1 : 10^3 (d) 1 : 10^4

20. A point performs simple harmonic oscillation of period T and the equation of motion is given by

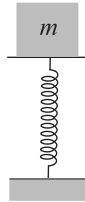
$x = a \sin\left(\omega t + \frac{\pi}{6}\right)$ the elapse of what fraction

of the time period, the velocity of the point will be equal to half of its maximum velocity ?

[AIPMT 2008]

- (a) $\frac{T}{8}$ (b) $\frac{T}{6}$
 (c) $\frac{T}{3}$ (d) $\frac{T}{12}$

21. A mass of 2.0 kg is put on a flat i pan attached to a vertical spring fixed on the ground as shown in 0 the figure. The mass of the spring g and the pan is negligible. When g pressed slightly and released the mass executes a simple harmonic motion. The spring constant is 200 N/m What should be the minimum amplitude of the motion, so that the mass gets detached from the pan? (Take $g = 10 \text{ m/s}^2$) [AIPMT 2007]



- (a) 8.0 cm
 (b) 10.0 cm
 (c) Any value less than 12.0 cm
 (d) 4.0 cm

22. The particle executing simple harmonic motion has a kinetic energy $K_0 \cos^2 \omega t$. The maximum values of the potential energy and the total energy are respectively [AIPMT 2007]

- (a) 0 and $2K_0$ (b) $\frac{K_0}{2}$ and K_0
 (c) K_0 and $2K_0$ (d) K_0 and K_0

23. A particle executes simple harmonic oscillation with an amplitude a. The period of oscillation is T. The minimum time taken by the particle to travel half of the amplitude from the equilibrium position is [AIPMT 2007]

- (a) $\frac{T}{4}$ (b) $\frac{T}{8}$
 (c) $\frac{T}{12}$ (d) $\frac{T}{2}$

24. A rectangular block of mass m and area of cross-section A floats in a liquid of density ρ . If it is given a small vertical displacement from equilibrium, it undergoes oscillation with a time period T. Then [AIPMT 2006]

- (a) $T \propto \sqrt{\rho}$ (b) $T \propto \frac{1}{\sqrt{A}}$

- (c) $T \propto \frac{1}{\rho}$ (d) $T \propto \frac{1}{\sqrt{m}}$

25. A particle executing simple harmonic motion of amplitude 5 cm has maximum speed of 31.4 cm/s. The frequency of its oscillation is [AIPMT 2005]

- (a) 3 Hz (b) 2 Hz
 (c) 4 Hz (d) 1 Hz

26. Two springs of spring constants k_1 and k_2 are joined in series. The effective spring constant of the combination is given by [AIPMT 2004]

- (a) $\sqrt{k_1 k_2}$ (b) $\frac{(k_1 + k_2)}{2}$
 (c) $k_1 + k_2$ (d) $\frac{k_1 k_2}{(k_1 + k_2)}$

27. Which one of the following statements is true for the speed v and the acceleration a of a particle executing simple harmonic motion? [AIPMT 2004]

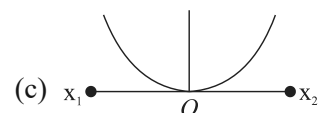
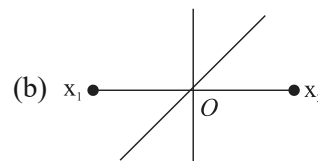
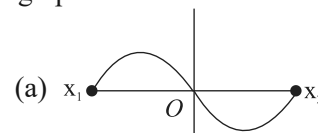
- (a) When v is maximum, a is maximum
 (b) Value of a is zero, whatever may be the value of y
 (c) When v is zero, a is zero
 (d) When v is maximum, a is zero

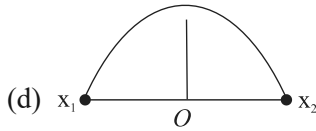
28. The potential energy of a simple harmonic oscillator when the particle is half way to its end point is [AIPMT 2003]

- (a) $\frac{1}{4}E$ (b) $\frac{1}{2}E$
 (c) $\frac{2}{3}E$ (d) $\frac{1}{8}E$

(where, E is the total energy)

29. A particle of mass m oscillates with simple harmonic motion between points x_1 and x_2 , the equilibrium position being O. Its potential energy is plotted. It will be as given below in the graph [AIPMT 2003]





- (d) x_1 x_2
O
30. The displacement of particle between maximum potential energy position and maximum kinetic energy position in simple harmonic motion is
[AIPMT 2002]

- (a) $\pm \frac{a}{2}$ (b) $\pm a$
(c) $\pm 2a$ (d) ± 1

31. When a damped harmonic oscillator completes 100 oscillations, its amplitude is I reduced to $\frac{1}{3}$ of its initial value. What will be its amplitude when it completes 200 oscillations?
[AIPMT 2002]

- (a) $\frac{1}{5}$ (b) $\frac{2}{3}$
(c) $\frac{1}{6}$ (d) $\frac{1}{9}$

32. A mass is suspended separately by two springs of spring constants k_1 and k_2 in a successive order. The time periods of oscillations in the two cases are T_1 and T_2 respectively. If the same mass be suspended by connecting the two springs in parallel, (as shown in figure) then the time period of oscillations is T . The correct relation is
[AIPMT 2002]

- (a) $T^2 = T_1^2 + T_2^2$ (b) $T^{-2} = T_1^{-2} + T_2^{-2}$
(c) $T^{-1} = T_1^{-1} + T_2^{-1}$ (d) $T = T_1 + T_2$

33. In SHM restoring force is $F = -kx$, where k is force constant, x is displacement and a is amplitude of motion, then total energy depends upon
[AIPMT 2001]

- (a) k, a and m (b) k, x, m
(c) k, a (d) k, x

34. Two simple harmonic motions given by, $x = a \sin(\omega t + \delta)$ and $y = a \sin\left(\omega t + \delta + \frac{\pi}{2}\right)$ act on a particle simultaneously, then the motion of particle will be
[AIPMT 2000]

- (a) circular anti-clockwise
(b) circular clockwise
(c) elliptical anti-clockwise
(d) elliptical clockwise

35. A pendulum is displaced to an angle θ from its equilibrium position, then it will pass through its mean position with a velocity v equal to
[AIPMT 2000]

- (a) $\sqrt{2gl}$ (b) $\sqrt{2gl \sin \theta}$
(c) $\sqrt{2gl \cos \theta}$ (d) $\sqrt{2g(l - \cos \theta)}$

JEE & AIEEE

1. Under an adiabatic process, the volume of an ideal gas gets doubled. Consequently the mean collision time between the gas molecules changes from τ_1 to τ_2 . If $\frac{C_p}{C_v} = \gamma$ for this gas then a good estimate for τ_1/τ_2 is given by
[JEE Mains 2020]

- (a) $\left(\frac{1}{2}\right)^\gamma$ (b) 2
(c) $\frac{1}{2}$ (d) $\left(\frac{1}{2}\right)^{\gamma+1}$

2. If a spring has time period T , and is into n equal parts, then the time period of each part will be
[AIEEE 2002]

- (a) $T\sqrt{n}$ (b) T/\sqrt{n}
(c) nT (d) T

3. A mass M is suspended from a spring of negligible mass. The spring is pulled a little and released so that the mass executes SHM of time period T . If the mass is increased by m , the time period becomes $5T/3$. then the ratio of m/M is
[AIEEE 2003]

- (a) $3/5$ (b) $25/9$
(c) $16/9$ (d) $5/3$

4. A particle of mass m is attached to a spring (of spring constant k) and has a natural angular frequency ω_0 . An external force $F(t)$ proportional to $\cos \omega t$ ($\omega \neq \omega_0$) is applied to the oscillator. The time displacement of the oscillator will be proportional to
[AIEEE 2004]

- (a) $\frac{m}{\omega_0^2 - \omega^2}$ (b) $\frac{1}{m(\omega_0^2 - \omega^2)}$
(c) $\frac{1}{m(\omega_0^2 + \omega^2)}$ (d) $\frac{m}{\omega_0^2 + \omega^2}$

5. Two simple harmonic motions are represented by the equation $y_1 = 0.1 \sin\left(100\pi t + \frac{\pi}{3}\right)$ and $y_2 = 0.1 \cos pt$. The phase difference of the velocity of particle 1 w.r.t. the velocity of the particle 2 is
[AIEEE 2005]

- (a) $-\pi/6$ (b) $\pi/3$

- (c) $-\pi/3$ (d) $\pi/6$

6. The maximum velocity of a particle, executing simple harmonic motion with an amplitude of 7 mm, is 4.4 m/s. The period of oscillation is

[AIEEE 2006]

- (a) 100 s (b) 0.01 s
(c) 10 s (d) 0.1 s

7. The displacement of an object attached to a spring and executing simple harmonic motion is given by $x = 2 \times 10^{-2} \cos \pi/\text{metre}$. The time at which the maximum speed first occurs is

[AIEEE 2007]

- (a) 0.5s (b) 0.75s
(c) 0.125s (d) 0.25s

8. If x , v and a denote the displacement, velocity and acceleration of a particle executing simple harmonic respectively and T represents the time period of the motion, then, which of the following does not change with time?

[AIEEE 2009]

- (a) $a^2T^2 + 4\pi^2v^2$ (b) $\frac{aT}{x}$
(c) $aT + 2\pi v$ (d) $\frac{aT}{v}$

9. A mass M , attached a horizontal spring, executes SHM with amplitude A_1 . When the M passes through its mean position then a smaller mass m is placed over it and both of them move together with amplitude A_2 . The ratio of $\left(\frac{A_1}{A_2}\right)$ is

[AIEEE 2011]

- (a) $\frac{M+m}{M}$ (b) $\left(\frac{M+m}{M}\right)^{1/2}$
(c) $\frac{M}{M+m}$ (d) $\frac{M}{M+m}$

10. An ideal gas enclosed in a vertical cylindrical container supports a freely moving piston of mass M . The piston and cylinder have equal cross sectional area A . When the piston is in equilibrium, the volume of the gas is V_0 and its pressure is P_0 . The piston is slightly displaced from the equilibrium position and released. Assuming that the system is completely isolate from its surrounding, the piston executes a simple harmonic motion with frequency

[JEE Mains 2013]

- (a) $\frac{1}{2\pi} \frac{V_0MP_0}{A^2\gamma}$ (b) $\frac{1}{2\pi} \sqrt{\frac{A^2\gamma P_0}{MV_0}}$

- (c) $\frac{1}{2\pi} \sqrt{\frac{MV_0}{A\gamma P_0}}$ (d) $\frac{1}{2\pi} \frac{A\gamma P_0}{MV_0}$

11. A particle performs simple harmonic motion with amplitude A . Its speed is trebled at the instant that it is at a distance $2A/3$ from equilibrium position. The new amplitude of the motion is

[JEE Mains 2016]

- (a) $\frac{7A}{3}$ (b) $\frac{A}{3}\sqrt{41}$
(c) $3A$ (d) $A\sqrt{3}$

12. In an experiment to determine the period of a simple pendulum of length 1 m, it is attached to difference spherical bobs of radii r_1 and r_2 . The two spherical bobs have uniform mass distribution. If the relative difference in the periods, is found to be 5×10^{-4} s, the difference in radii, $|r_1 - r_2|$ is best given be

[JEE Mains 2017]

- (a) 0.5 cm (b) 0.01 cm
(c) 0.1 cm (d) 1 cm

NEET & AIPMT Answer Key

- | | | |
|-------|-------|-------|
| 1. B | 2. D | 3. B |
| 4. B | 5. C | 6. D |
| 7. C | 8. B | 9. D |
| 10. B | 11. D | 12. C |
| 13. C | 14. B | 15. C |
| 16. D | 17. B | 18. D |
| 19. B | 20. D | 21. B |
| 22. D | 23. C | 24. B |
| 25. D | 26. D | 27. D |
| 28. A | 29. C | 30. B |
| 31. D | 32. B | 33. C |
| 34. B | 35. D | |

JEE Answer Key

- | | | |
|-------|-------|-------|
| 1. D | 2. B | 3. C |
| 4. B | 5. A | 6. C |
| 7. A | 8. B | 9. C |
| 10. B | 11. A | 12. C |