7 & 8

Oscillations & Waves

Oscillations

Periodic motion - time period, frequency, displacement as a function of time. Periodic functions.
Simple harmonic motion (S.H.M) and its equation; phase; oscillations of a spring-restoring force and force constant; energy in S.H.M. Kinetic and potential energies; simple pendulum derivation of expression for its time period.
Free, forced and damped oscillations (qualitative ideas only), resonance.

Waves

Wave motion. Transverse and longitudinal waves, speed of wave motion. Displacement relation for a progressive wave. Principle of superposition of waves, reflection of waves, standing waves in strings and organ pipes, fundamental mode and harmonics, Beats, Doppler effect.

MIND MAP : LEARNING MADE SIMPLE

Oscillations

- Every oscillatory motion is periodic, but every periodic motion need not be oscillatory.
- To and fro motion repeatedly about a fixed point in a definite interval of time.

SHM

Simple Harmonic Motion is the simplest form of oscillatory motion

Displacement in SHM

$x(t) = A(\cos\omega t + \phi)$

A = Amplitude i.e., maximum displacement of particles.

Phase

Time varying quantity $(\omega t + \phi)$

- Phase constant or Phase angle (ϕ)
- It depends upon velocity (v) and displacement of particle at $t = 0$

Time period (T)

The smallest interval of time after which the motion is repeated

Frequency

The number of oscillations per second

i.e., $f = \frac{1}{T} = \frac{\omega}{2\pi}$

Velocity in SHM

$v = \frac{dx}{d(t)} = -\omega A \sin(\omega t + \varphi)$

$v_{max} = \omega A$

Acceleration in SHM

$a = \frac{dv}{dt} = -\omega^2 A \cos(\omega t + \varphi)$

$a_{max} = \omega^2 A$

Energy in SHM

Kinetic Energy

$K.E = \frac{1}{2} m\omega^2 A^2 \sin^2(\omega t + \varphi)$

Potential Energy

$P.E = \frac{1}{2} m\omega^2 A^2 \cos^2(\omega t + \varphi)$

Total energy, T.E. = K.E. + P.E.

$= \frac{1}{2} kA^2[\cos^2(\omega t + \varphi) + \sin^2(\omega t + \varphi)]$

Force law equation for SHM

$F = -k(x), k = m\omega^2$

$\therefore \omega = \sqrt{\frac{k}{m}}$

Simple pendulum

Time period $T = 2\pi\sqrt{\frac{I}{mgl}} = 2\pi\sqrt{\frac{l}{g}}$

Oscillations due to a spring

Time period $T = 2\pi\sqrt{\frac{m}{k}}$

Here, $k = \frac{F}{x}$ = *spring constant*

Damped Oscillations

Oscillation of a body whose amplitude goes on decreasing with time.

Damping force $F_d = -bv$, b = damping constant

Displacement of damped oscillator

$$x = x_m e^{-bt/2m} \sin(\omega t + \varphi)$$

Amplitude decreases with time as

$$x = x_m e^{-(b/2m)t}$$

Forced Oscillations

Driving force, $F(t) = F_o \cos\omega_d t$

Displacement, $x(t) = A\cos(\omega_d + \varphi)$

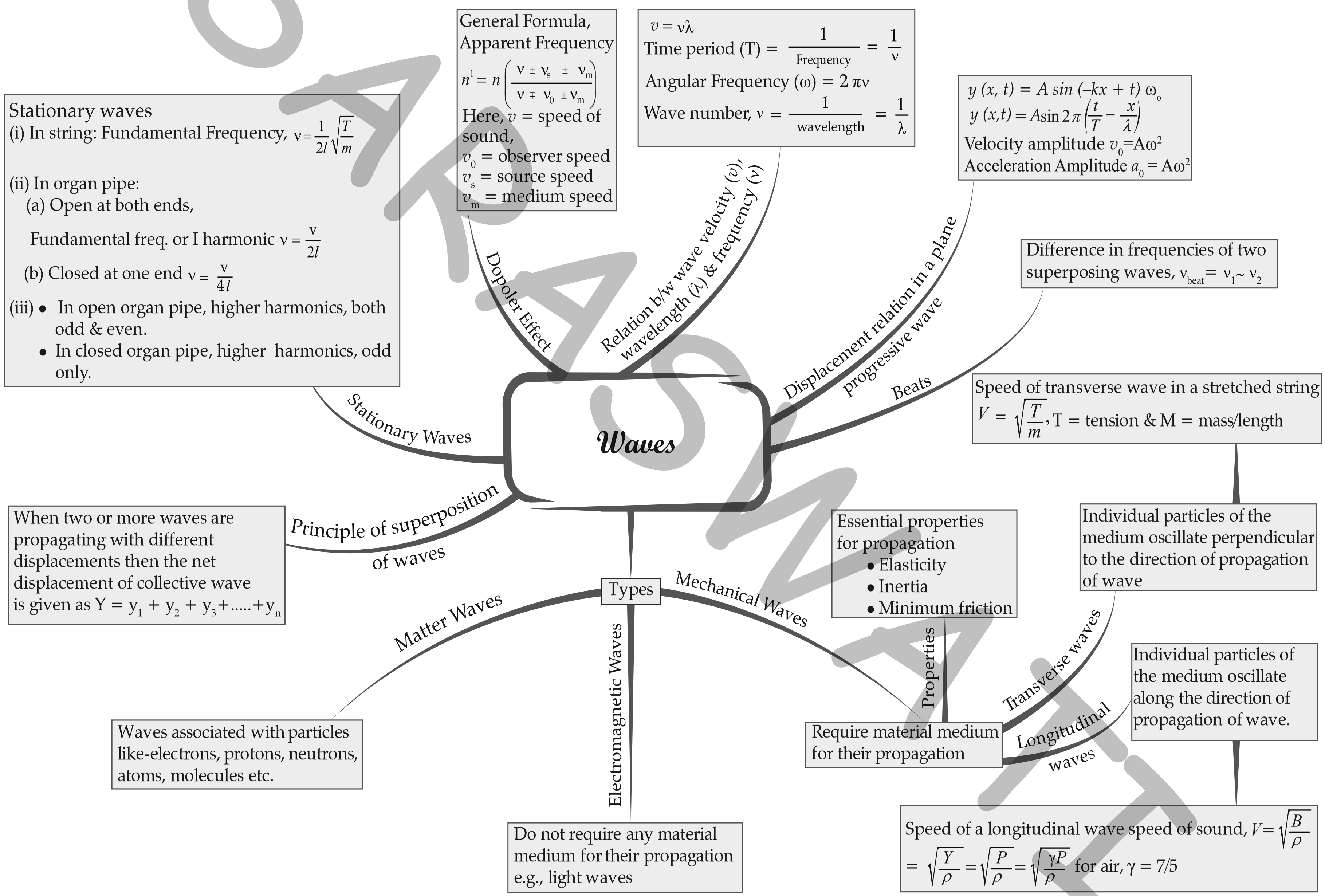
Amplitude, $A = \frac{F_o}{\left\{m^2\left(\omega^2 - \omega_d^2\right)^2 + \omega_d^2 b^2\right\}^{\frac{1}{2}}}$

$\tan\varphi = \frac{-v_o}{\omega_d x_o}$

Resonance

When frequency of external force is equal to natural frequency of oscillator.

MIND MAP : LEARNING MADE SIMPLE



OSCILLATION & WAVES

1. INTRODUCTION

(1) A motion which repeats itself over and over again after a regular interval of time is called a periodic motion.

(2) Oscillatory or vibratory motion is that motion in which a body moves to and fro or back and forth repeatedly about a fixed point in a definite interval of time.

(3) Simple harmonic motion is a specific type of oscillatory motion, in which

(a) partical moves in one dimension,

(b) particle moves to and fro about a fixed mean position (where $F_{net} = 0$),

(c) net force on the particle is always directed towards means position, and

(d) magnitude of net force is always proportional to the displacement of particle from the mean position at that instant.

So, $F_{net} = -kx$

where, k is known as force constant

$\Rightarrow$ $ma = -kx$

$\Rightarrow$ $a = \dfrac{-k}{m}x$ or $a = -\omega^2 x$

where, ω is known as angular frequency.

$$\Rightarrow \quad \frac{d^2x}{dt^2} = -\omega^2 x$$

This equation is called as the differential equation of S.H.M.

The general expression for x(t) satisfying the above equation is :

$x(t) = A\sin(\omega t + \phi)$

1.1 Some Important terms

1. Amplitude

The amplitude of particle executing S.H.M. is its maximum displacement on either side of the mean position.

A is the amplitude of the particle.

2. Time Period

Time period of a particle executing S.H.M. is the time taken to complete one cycle and is denoted by T.

Time period $(T) = \dfrac{2\pi}{\omega} = 2\pi\sqrt{\dfrac{m}{k}}$ as $\omega = \sqrt{\dfrac{m}{k}}$

3. Frequency

The frequency of a particle executing S.H.M. is equal to the number of oscillations completed in one second.

$$v = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

4. Phase

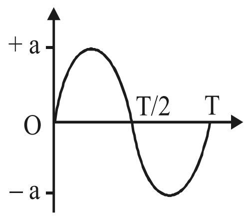
The phase of particle executing S.H.M. at any instant is its state as regard to its position and direction of motion at that instant. it is measured as argument (angle) of sine in the equation of S.H.M.

Phase $= (\omega t + \phi)$

At $t = 0$, phase $= \phi$; the constant ϕ is called initial phase of the particle or phase constant.

1.2 Important Relations

1. Position

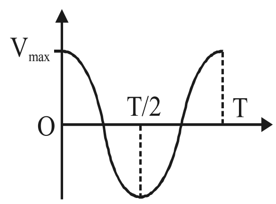


If mean position is at origin the position (X coordinate) depends on time in general as :

$x(t) = \sin(\omega t + \phi)$

- At mean position, $x = 0$
- At extremes, $x = +A, -A$

2. Velocity



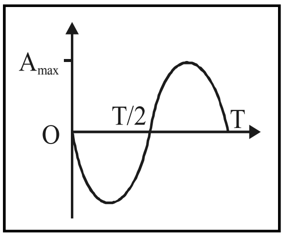
- At any time instant t, $v(t) = A\omega\cos(\omega t + \phi)$

- At any position x, $v(x) = \pm\, \omega\sqrt{A^2 - x^2}$
- Velocity is minimum at extremes because the particles is at rest.

  i.e., $v = 0$ at extreme position.
- Velocity has maximum magnitude at mean position.

  $|v|_{max} = \omega A$ at mean position.

3. Acceleration



- At any instant t, $a(t) = -\omega^2 A \sin(\omega t + \phi)$
- At any position x, $a(x) = -\omega^2 x$
- Acceleration is always directed towards mean position.
- The magnitude of acceleration is minimum at mean position and maximum at extremes.

  $|a|_{min} = 0$ at mean position.

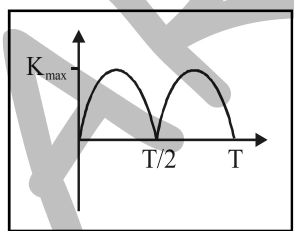
  $|a|_{max} = \omega^2 A$ at extremes.

4. Energy

Kinetic energy

- $K = \frac{1}{2}mv^2 \Rightarrow K = \frac{1}{2}m\omega^2\left(A^2 - x^2\right)$

  $= \frac{1}{2}m\omega^2 A^2 \cos^2\left(\omega t + \phi\right)$

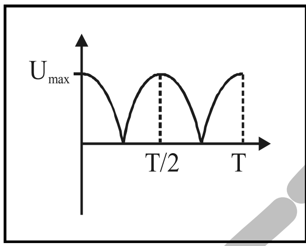


- K is maximum at mean position and minimum at extremes.
- $K_{max} = \frac{1}{2}m\,\omega^2 A^2 = \frac{1}{2}kA^2$ at mean position
- $K_{min} = 0$ at extemes.

Potential Energy

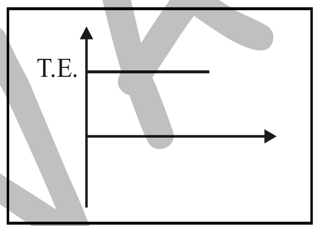
If potential energy is taken as zero at mean position, then at any position x,

$$U(x) = \frac{1}{2}kx^2 = \frac{1}{2}mA^2\omega^2 \sin^2\left(\omega t + \phi\right)$$



- U is maximum at extremes $U_{max} = \frac{1}{2}kA^2$
- U is minimum at mean position

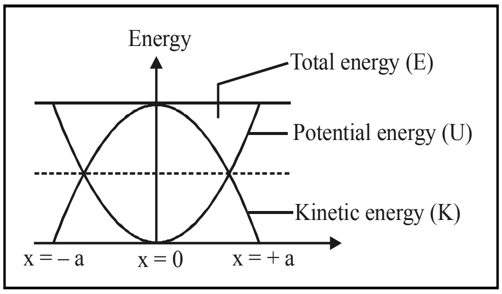
Total Energy



$$T.E. = \frac{1}{2}kA^2 = \frac{1}{2}mA^2\omega^2$$

and is constant at all time instant and at all positions.

Energy position graph



2. TIME PERIOD OF S.H.M.

To find whether a motion is S.H.M. or not and to find its time period, follow these steps :

(a) Locate the mean (equilibrium) position mathematically by balancing all the forces on it.

(b) Displace the particle by a displacement 'x' from the mean position in the probable direction of oscillation.

(c) Find the net force on it and check if it is towards mean position.

(d) Try to express net force as a proportional function of its displacement 'x'.

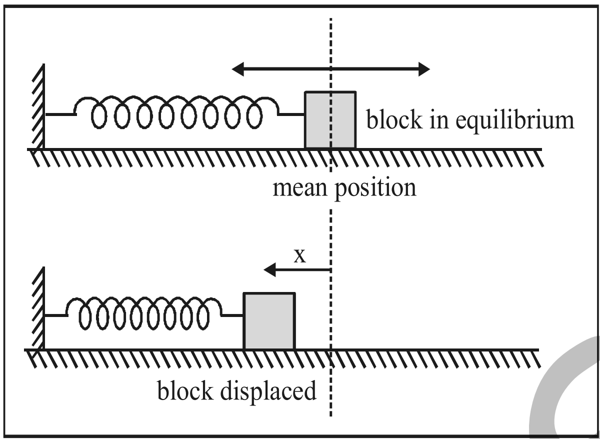
- If step (c) and step (d) are proved then it is a simple harmonic motion.

(e) Find k from expression of net force ($F = -kx$) and find time period using $T = 2\pi\sqrt{\frac{m}{k}}$.

2.1 Oscillations of a Block Connected to a Spring

(a) Horizontal spring :

Let a block of mass m be placed on a smooth horizontal surface and rigidly connected to spring of force constant K whose other end is permanently fixed.

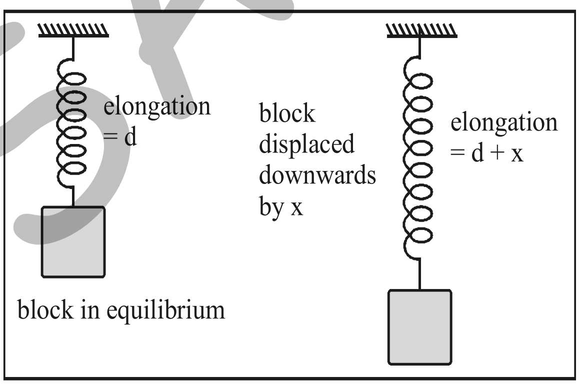


- Mean position : when spring is at its natural length.

- Time period : $T = 2\pi\sqrt{\frac{m}{k}}$

(b) Vertical Spring :

If the spring is suspended vertically from a fixed point and carries the block at its other end as shown, the block will oscillate along the vertical line.



- Mean position : spring in elongated by $d = \frac{mg}{k}$

- Time period : $T = 2\pi\sqrt{\frac{m}{k}}$

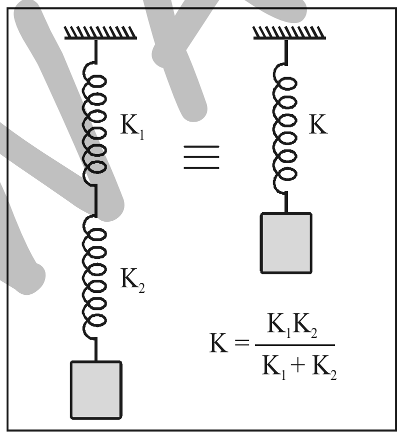
(c) Combination of springs :

1. Springs in series

When two springs of force constant K_1 and K_2 are connected in series as shown, they are equivalent to a single spring of force constant K which is given by

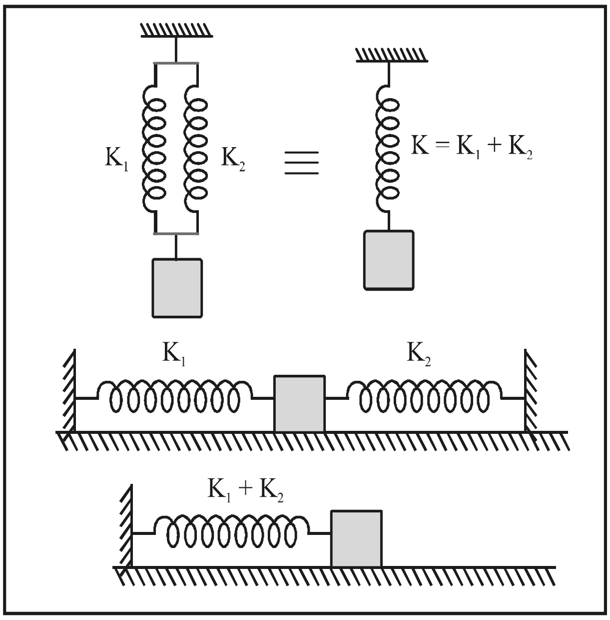
$$\frac{1}{K} = \frac{1}{K_1} + \frac{1}{K_2}$$

$$K = \frac{K_1K_2}{K_1 + K_2}$$



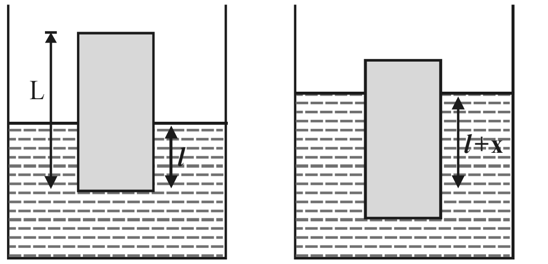
2. Springs in parallel

For a parallel combination as shown, the effective spring constant is $K = K_1 + K_2$



2.2 Oscillation of a Cylinder Floating in a liquid

Let a cylinder of mass m and density d be floating on the surface of a liquid of density ρ. The total length of cylinder is L.

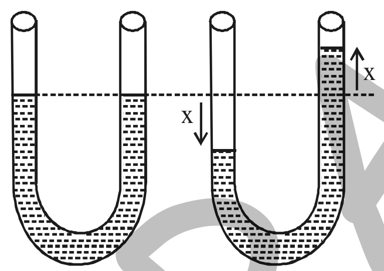


cylinder in equilibrium   cylinder displaced downwards by a distance x

- Mean position : cylinder is immersed upto $\ell = \dfrac{Ld}{\rho}$
- Time period : $T = 2\pi\sqrt{\dfrac{Ld}{\rho g}} = 2\pi\sqrt{\dfrac{\ell}{g}}$

2.3 Liquid Oscillating in a U–Tube

Consider a liquid column of mass m and density ρ in a U-tube of area of cross section A.

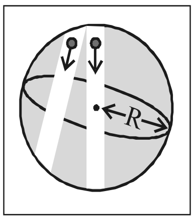


In equilibrium   liquid displaced by x

- Mean position : when height of liquid is same in both limbs.
- Time period : $T = 2\pi\sqrt{\dfrac{m}{2A\rho g}} = 2\pi\sqrt{\dfrac{L}{2g}}$

  where, L is length of liquid column.

2.4 Body Oscillation in tunnel along any chord of earth



- Mean position : At the centre of the chord
- Time period : $T = 2\pi\sqrt{\dfrac{R}{g}} = 84.6$ minutes

  where, R is radius of earth.

2.5 Angular Oscillations

Instead of straight line motion, if a particle or centre of mass of a body is oscillating on a small arc of circular path then it is called angular S.H.M.

For angular S.H.M., $\tau = -k\theta$

$\Rightarrow$ $I\alpha = -k\theta$

$\Rightarrow$ Time period, $T = 2\pi\sqrt{\dfrac{I}{k}}$

2.5.1 Simple Pendulum

- Time period : $T = 2\pi\sqrt{\dfrac{\ell}{g}}$
- Time period of a pendulum in a lift :

  $T = 2\pi\sqrt{\dfrac{\ell}{g+a}}$ (if acceleration of lift is upwards)

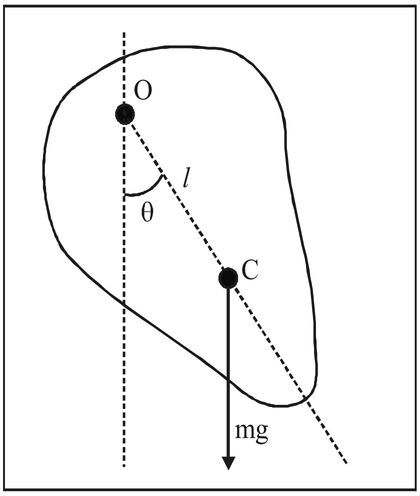
  $T = 2\pi\sqrt{\dfrac{\ell}{g-a}}$ (if acceleration of lift is downwards)
- Second's pendulum

  Time period of second's pendulum is 2s.

  Length of second's pendulum on earth surface $\simeq$ 1m.

2.5.2 Physical Pendulum

Time period : $T = 2\pi\sqrt{\dfrac{I}{mg\ell}}$



where, I is moment of inertia of object about point of suspension, and

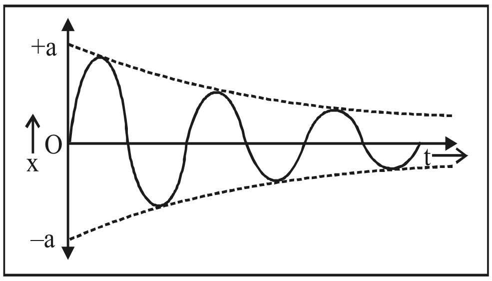
l is distance of centre of mass of object from point of suspension.

3. DAMPED AND FORCED OSCILLATIONS

1. Damped Oscillation :

(i) The oscillation of a body whose amplitude goes on decreasing with time is defined as damped oscillation.

(ii) In this oscillation the amplitude of oscillation decreases exponentially due to damping forces like frictional force, viscous force etc.



(iii) Due to decrease in amplitude the energy of the oscillator also goes on decreasing exponentially.

2. Forced Oscillation :

(i) The oscillation in which a body oscillates under the influence of an external periodic force are known as forced oscillation.

(ii) Resonance : When the frequency of external force is equal to the natural frequency of the oscillator, then this state is known as the state of resonance. And this frequency is known as resonant frequency.

4. WAVES

(a) Speed of longitudinal wave

- Speed of longitudinal wave in a medium is given by

$$v = \sqrt{\frac{E}{\rho}}$$

where, E is the modulus of elasticity,

ρ is the density of the medium.

- Speed of longitudinal wave in a solid in the form of rod is given by

$$v = \sqrt{\frac{Y}{\rho}}$$

where, Y is the Young's modulus of the solid,

ρ is the density of the solid.

- Speed of longitudinal wave in fluid is given by

$$v = \sqrt{\frac{B}{\rho}}$$

where, B is the bulk modulus,

ρ is the density of the fluid.

(b) Newton's formula

- Newton assumed that propagation of sound wave in gas is an isothermal process. Therefore, according to Newton, speed of sound in gas is given by $v = \sqrt{\frac{P}{\rho}}$

where P is the pressure of the gas and ρ is the density of the gas.

- According to the Newton's formula, the speed of sound in air at S.T.P. is 280 m/s. But the experimental value of the speed of sound in air is 332 m s^{-1}. Newton could not explain this large difference. Newton's formula was corrected by Laplace.

(c) Laplace's correction

- Laplace assumed that propagation of sound wave in gas in an adiabatic process. Therefore, according to Laplace, speed of sound in a gas is given by

$$v = \sqrt{\frac{\gamma P}{\rho}}$$

- According to Laplace's correction the speed of sound in air at S.T.P. is 331.3 m/s. This value agrees farily well with the experimental values of the velocity of sound in air at S.T.P.

5. WAVES TRAVELLING IN OPPOSITE DIRECTIONS

- When two waves of same amplitude and frequency travelling in opposite directions

$$y_1 = A \sin (kx - \omega t)$$

$$y_2 = A \sin (kx + \omega t)$$

interfere, then a standing wave is produced which is given by,

$$y = y_1 + y_2$$

$$\Rightarrow \; y = 2A \sin kx \cos \omega t$$

- Hence the particle at location x is oscillating in S.H.M. with angular frequency ω and amplitude 2A sin kx. As the amplitude depends on location (x), particles are oscillating with different amplitude.

- **Nodes :** Amplitude = 0

  $2A \sin kx = 0$

  $x = 0, \pi/k, 2\pi/k.......$

  $x = 0, \lambda/2, \lambda, 3\lambda/2, 2\lambda........$

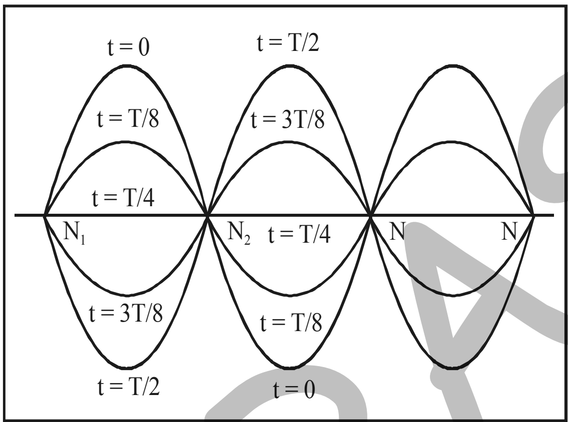
- **Antinodes :** Amplitude is maximum.

  $\sin kx = \pm 1$

  $x = \pi/2k, 3\pi/2k$

  $x = \lambda/4, 3\lambda/4, 5\lambda/4$

- Nodes are completely at rest. Antinodes are oscillating with maximum amplitude (2A). The points between a node and antinode have amplitude between 0 and 2 A.
- Separation between two consecutive (or antinodes) $= \lambda/2$.
- Separation between a node and the next antinode$=\lambda/4$.
- Nodes and antinodes are alternately placed.



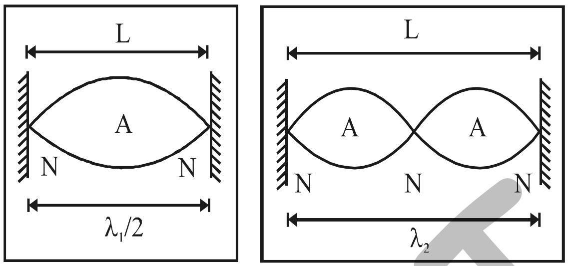
- It is clear from the figure that since nodes are, at rest they don't transfer energy. In a stationary wave, energy is not transferred from one point to the other.

5.1 Vibrations in a stretched string

1. Fixed at both ends.

- Transverse standing waves with nodes at both ends of the string are formed.
- So, length of string, $\ell = \frac{n\lambda}{2}$ if there are (n + 1) nodes and n antinodes.
- Frequency of oscillations is

$$\Rightarrow \nu = \frac{v}{\lambda} = \frac{nv}{2\ell}$$



- Fundamental frequency (x = 1)

$$\nu_0 = \frac{v}{2L}$$

It is also called first harmonic.

- Second harmonic or first overtone

$$\nu = \frac{2v}{2L}$$

- The nth multiple of fundamental frequency is known as nth harmonic or (n – 1)th overtone.

2. Fixed at one end

- Transverse standing waves with node at fixed end and antinode at open end are formed.
- So, length of string $\ell = (2n-1)\frac{\lambda}{4}$ if there are n nodes and n antinodes.
- Frequency of oscillations

$$\Rightarrow \nu = \frac{v}{\lambda} = \frac{(2n-1)v}{4\ell}$$

- Fundamental frequency, (n = 1)

$$\nu_0 = \frac{v}{4L}$$

It is also called first harmonic.

- First overtone or third harmonic.

$$\nu = \frac{3v}{4\ell} = 3\nu_0$$

- Only odd harmonics are possible in this case.

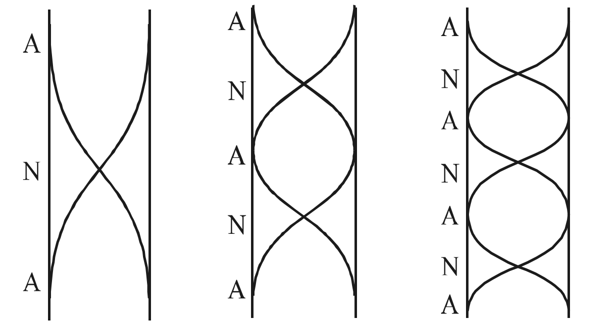
5.2 Vibrations in an organ pipe

1. Open Organ pipe (both ends open)

- The open ends of the tube becomes antinodes because the particles at the open end can oscillate freely.
- If there are (n + 1) antinodes in all,

length of tube, $\ell = \frac{n\lambda}{2}$

- So, Frequency of oscillations is $\nu = \frac{nv}{2\ell}$



fundamental mode first harmonic | Second harmonic (first overtone) | Third harmonic (second overtone)

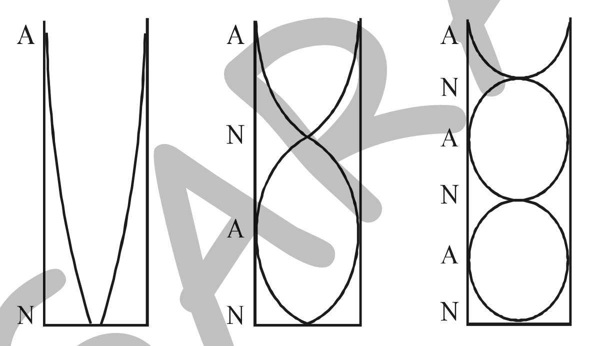
2. Closed organ pipe (One end closed)

- The open end becomes antinode and closed end become a node.
- If there are n nodes and n antinodes,

  $L = (2n - 1)\lambda/4$

- So frequency of oscillations is

$$\nu = \frac{v}{\lambda} = \frac{(2n-1)v}{4L}$$



fundamental mode first harmonic | third harmonic (first overtone) | fifth harmonic (second overtone)

- There are only odd harmonics in a tube closed at one end.

5.3 Waves having different frequencies

Beats are formed by the superposition of two waves of slightly different frequencies moving in the same direction. The resultant effect heard in this case at any fixed position will consist of alternate loud and weak sounds.

Let us consider net effect of two waves of frequencies ν_1 and ν_2 and amplitude A at x = 0.

$$y_1 = A\sin 2\pi\nu_1 t$$

$$y_2 = A\sin 2\pi\nu_2 t$$

$$\Rightarrow y = y_1 + y_2$$

- $\Rightarrow y = A\left(\sin 2\pi\nu_1 t + \sin 2\pi\nu_2 t\right)$

$$y = \left[2A\cos\pi\left(\nu_1 - \nu_2\right)t\right]\sin\pi\left(\nu_1 + \nu_2\right)t$$

Thus the resultant wave can be represented as a travelling wave whose frequency is $\left(\frac{\nu_1 + \nu_2}{2}\right)$ and amplitude is $2A\cos\pi(\nu_1 - \nu_2)t$.

As the amplitude term contains t, the amplitude varies periodically with time.

For Loud Sounds : Net amplitude = ± 2A

$\Rightarrow \cos\pi(\nu_1 - \nu_2)t = \pm 1$

$\Rightarrow \pi(\nu_1 - \nu_2)t = 0, \pi, 2\pi, 3\pi$ ....

$$\Rightarrow t = 0, \frac{1}{\nu_1 - \nu_2}, \frac{2}{\nu_1 - \nu_2} \text{........}$$

Hence the interval between two loud sounds is given as :

$$= \frac{1}{\nu_1 - \nu_2}$$

$\Rightarrow$ the number of loud sounds per second = $\nu_1 - \nu_2$

$\Rightarrow$ beat per second = $\nu_1 - \nu_2$

Note that $\nu_1 - \nu_2$ must be small (0 – 16 Hz) so that sound variations can be distinguished.

Note..

- Filling a tuning form increases its frequency of vibration.
- Loading a tuning for k decreases its frequency of vibration.

6. DOPPLER EFFECT

According to Doppler's effect, whenever there is a relative motion between a source of sound and listener, the apparent frequency of sound heard by the listener is different from the actual frequency of sound emitted by the source.

Apparent frequency,

$$\boxed{\nu' = \frac{v - v_L}{v - v_s} \times \nu}$$

Sing Convention. All velocities along the direction S to L are taken as positve and all velocities along the direction L to S are taken as negative.

When the motion is along some other direction the component of velocity of source and listener along the line joining the source and listener is considered.

Special Cases :

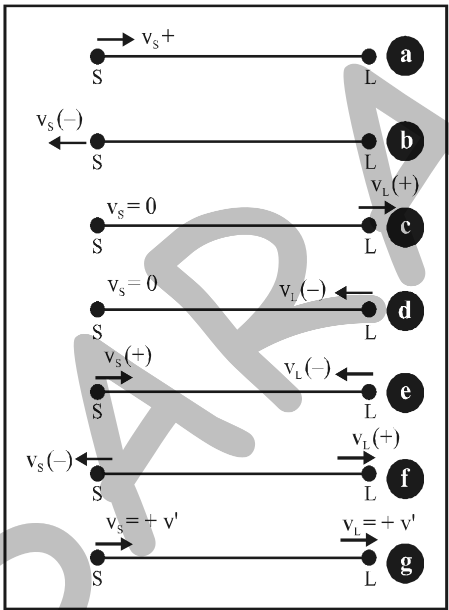
(a) If the source is moving towards the listener but the listener is at rest, then v_s is positive and $v_L = 0$ (figure a). Therefore,

$$\boxed{\nu' = \frac{v}{v - v_s} \times \nu} \quad \text{i.e. } \nu' > \nu$$

(b) If the source is moving away from the listener, but the listener is at rest, then v_s is negative and $v_L = 0$ (figure b). Therefore,

$$\boxed{\nu' = \frac{v}{v-(-v_s)}\nu = \frac{v}{v+v_s}\nu} \quad \text{i.e. } \nu' < \nu$$

(c) If the source is at rest and listener is moving away from the source, the $v_s = 0$ and v_L is positive (figure c). Therefore,



$$\boxed{\nu' = \frac{(v - v_L)}{v}\nu} \quad \text{i.e. } \nu' < \nu$$

(d) If the source is at rest and listener is moving towards the source, then $v_s = 0$ and v_L is negative (figure d). Therefore,

$$\boxed{\nu' = \frac{v-(-v_L)}{v}\nu = \frac{v+v_L}{v}\nu} \quad \text{i.e. } \nu' > \nu$$

(e) If the source and listener are approaching each other, then v_s is positive and v_L is negative (figure e). Therefore,

$$\boxed{\nu' = \frac{v-(-v_L)}{v-v_s}\nu = \left(\frac{v+v_L}{v-v_s}\right)\nu} \quad \text{i.e. } \nu' > \nu$$

(f) If the source and listener are moving away from each other, then v_s is negative and v_L is positive, (figure f). Therefore,

$$\boxed{\nu' = \frac{v-v_L}{v-(-v_s)}\nu = \frac{v-v_L}{v+v_s}\nu} \quad \text{i.e. } \nu' < \nu$$

(g) If the source and listener are both in motion in the same direction and with same velocity, then $v_s = v_L = v'$ (say) (figure g). Therefore,

$$\boxed{\nu' = \frac{(v-v')}{(v-v')}\nu} \quad \text{i.e. } \nu' = \nu$$

It means, there is no change in the frequency of sound heard by the listerner.

Apparent wavelength heard by the observer is

$$\lambda' = \frac{v - v_s}{\nu}$$

Note...

If case the medium is also moving, the speed of sound with respect to ground is considered. i.e. $\vec{v} + \vec{v}_m$

7. CHARACTERISTICS OF SOUND

- **Loudness** of sound is also called level of intensity of sound.

  In decibel the loudness of a sound of intensity I is given by $L = 10 \log_{10}\left(\frac{I}{I_0}\right)$. ($I_0 = 10^{-12}$ w/m^2)

- **Pitch** : It is pitch depends on frequency, higher the frequency higher will be the pitch and shriller will be the sound.

QUESTION ALIKE
(Oscillations)

Very Short Answer Type Questions :

1. Can the motion of Moon around the Earth be taken as S.H.M.?

Sol. No, it is a circular and periodic motion but not to and fro about a mean position, which is essential for SHM.

2. Is the condition- acceleration proportional to displacement sufficient for S.H.M. and why?

Sol. The condition is not sufficient as it gives no reference to the direction of acceleration, whereas in S.H.M, the acceleration is always in a direction opposite to that of displacement.

3. A vibrating simple pendulum of period T is placed in a lift which is accelerating downwards. What will be the effect on time period?

Sol. Effective value of acceleration due to gravity decreases

$g' = g - a$ and $T \propto \frac{l}{\sqrt{g}}$. so, time period increases.

4. During the oscillation of a simple pendulum, what is the quantity that remains constant?

Sol. Total energy of bob in simple pendulum remains constant.

5. Is the oscillation of a mass suspended by a spring S.H.M.?

Sol. Yes, as it is periodic as well as oscillatory.

6. A spring mass system is made to oscillate horizontally and then vertically. What will be the change in period?

Sol. The time period remains same.

7. Sometimes, the body of an automobile begins to rattle when it picks up speed. Why?

Sol. This is due to resonant vibrations.

8. A body has maximum velocity in mean position and zero velocity at extreme position. Is it a sure test for SHM?

Sol. No

9. Can we conduct a simple pendulum experiment in an artificial satellite?

Sol. No, as there exists a state of weightlessness in an artificial satellite.

10. A simple pendulum is transferred from the Earth to the Jupiter. Will it go faster or slower?

Sol. Due to increase in value of g, time period shall decrease. So, the pendulum will vibrate faster.

Short Answer Type Questions :

11. Two identical pendulums are oscillating with amplitudes 1 cm and 2 cm. Calculate the ratio of their energies of oscillation.

Sol. Total energy of the bob of a simple pendulum is given by $E = \frac{1}{2}m\omega^2 a^2$ *i.e.* $E \propto a^2$

$$\Rightarrow \quad \frac{E_1}{E_2} = \frac{a_1^2}{a_2^2} = \left(\frac{1}{2}\right)^2 = \frac{1}{4}.$$

12. An SHM of amplitude x has a time period T. What will be the acceleration of the oscillator when its displacement is one third of the amplitude?

Sol. Acceleration, $a = -\omega^2 x = -\frac{4\pi^2}{T^2} \times \frac{x}{3} = -\frac{4\pi^2 x}{3T^2}$

13. Show that the motion of a body represented by $y = \sin\omega t - \cos\omega t$ is SHM with a period of $\frac{2\pi}{\omega}$.

Sol. $y = \sin\omega t - \cos\omega t$

$$= \sin\omega t + \sin\left(\omega t - \frac{\pi}{2}\right)$$

$$= 2\sin\left(\frac{\omega t + \omega t - \frac{\pi}{2}}{2}\right)\cos\left(\frac{\omega t - \omega t + \frac{\pi}{2}}{2}\right)$$

$$= 2\sin\left(\omega t - \frac{\pi}{4}\right)\cos\frac{\pi}{4}$$

$$= \frac{2}{\sqrt{2}}\sin\left(\omega t - \frac{\pi}{4}\right) = \sqrt{2}\sin\left(\omega t - \frac{\pi}{4}\right)$$

$\therefore$ time period, $T = \frac{2\pi}{\omega}$

14. Find the displacement of a simple harmonic oscillator at which its P.E. is half of the maximum energy of the oscillator.

Sol. Maximum energy, $E = \frac{1}{2}m\omega^2 r^2$ where r is the amplitude

PE at a distance x from the mean position is

$$PE = \frac{1}{2}m\omega^2 x^2 = \frac{1}{2}E$$

$$\therefore \quad \frac{1}{2}m\omega^2 x^2 = \frac{1}{4}m\omega^2 r^2$$

or $x = \pm\frac{r}{\sqrt{2}}$

15. What are the two basic characteristics of an SHM?

Sol. (i) Acceleration of a particle executing S.H.M. is directly proportional to the displacement of the particle. (ii) Acceleration and displacement of the particle are directed in opposite directions.

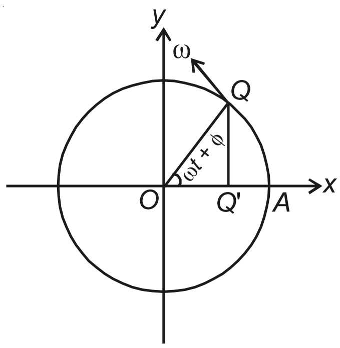
16. A mass of 4 kg is attached to the spring of spring constant 100 Nm^{-1}. The block is pulled to a distance of 10 cm from its equilibrium position at $x = 0$ on a horizontal frictionless surface from rest at $t = 0$. Write the expression for its displacement at any time t.

Sol. Amplitude, $r = 10$ cm $= 10 \times 10^{-2}$ m

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{100}{4}} \; 5 \text{ rad s}^{-1}$$

Using $x = r \sin\omega t$

$$x = \left(10 \times 10^{-2} \sin 5t\right) \text{m}$$

17. 

In figure what will be the sign of velocity of the point Q', which is the projection of the velocity of the reference particle Q. Q is moving in a circle of radius R in anticlockwise direction.

Sol. As the particle moves from A to Q in anticlockwise direction, the velocity of the projection is towards O *i.e.,* along negative X-axis. So, sign of the velocity will be negative.

18. The periodic time of a mass suspended by a spring (force constant k) is T. The spring is cut in two equal pieces. Same mass is suspended from one piece. What will be the periodic time?

Sol. Consider the spring be made of combination of two springs in series each of spring constant k. The effective spring constant k is given by

$$\frac{1}{K} = \frac{1}{k} + \frac{1}{k} = \frac{2}{k} \text{ or } k = 2K$$

Time period of vibration of a body attached to the end of this spring,

$$T = 2\pi\sqrt{\frac{m}{K}} = 2\pi\sqrt{\frac{m}{(k/2)}} = 2\pi\sqrt{\frac{2m}{k}} \qquad \text{...(i)}$$

When the spring is cut into two pieces, the spring constant = k. Time period of vibration of a body attached to the end of this spring, $T' = 2\pi\sqrt{\frac{m}{k}}$ ...(ii)

from (i) and (ii) $\frac{T'}{T} = \frac{1}{\sqrt{2}}$; or $T' = \frac{T}{\sqrt{2}}$

19. What will happen to the motion of a simple pendulum if the amplitude of motion is made large? How does period of oscillation change?

Sol. If amplitude of motion is made large, θ is large. In that case, $\sin\theta \neq \theta$, restoring torque will not be linear, so motion will not remain SHM but will become oscillatory. If angular amplitude is θ_0, time period will be given as

$$T = 2\pi\sqrt{\frac{L}{g}}\left(1+\frac{1}{2^2}\sin^2\frac{\theta_0}{2}+...\right)$$

$$= 2\pi\sqrt{\frac{L}{g}}\left(1+\frac{\theta_0^2}{16}\right)$$

20. The angular velocity and amplitude of a simple pendulum are ω and a respectively. At a displacement x from the mean position, if K.E is K and P.E. is U. Find $\frac{K}{U}$.

Sol. $\text{KE}, K = \frac{1}{2}mv^2 = \frac{1}{2}m\omega^2(a^2 - x^2)$

PE, $U = \frac{1}{2}m\omega^2 x^2$

so $\frac{K}{U} = \frac{a^2 - x^2}{x^2}$

21. Calculate the length of a second's pendulum.

Sol. $T = 2\pi\sqrt{\frac{L}{g}}$

$$L = \frac{gT^2}{4\pi^2} = \frac{9.8\times 2^2}{4\times\left(\frac{22}{7}\right)^2} = 0.99\,\text{m}.$$

22. The amplitude of an oscillating simple pendulum is 5 cm and its period 2 second. What will be its speed, 1 second after it passes its equilibrium position?

Sol. $a = 5$ cm, $T = 2$ s, $t = 1$ s, $v = ?$

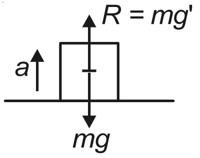
$v = a\omega\cos\omega t$

$= a\frac{2\pi}{T}\cos\frac{2\pi}{T}t$

$= \frac{5\times 2\times\pi}{2}\cos\frac{2\pi\times 1}{2}$

$= -5\pi$ cm/s

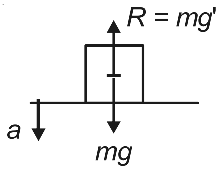
23. A vibrating simple pendulum of time period T is placed in a lift. What will be the effect on the time period when it accelerates (a) upwards (b) downwards?

Sol. (a) 

$R - Mg = Ma \qquad R = M(g + a)$

$g' = g + a$

$T = 2\pi\sqrt{\dfrac{L}{g'}} \qquad T \propto \dfrac{1}{\sqrt{g}},$ so, T decreases.

(b) 

$mg - R = ma \qquad R = m(g - a) \qquad g' = g - a$

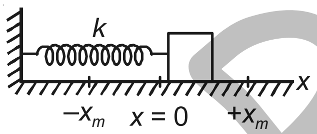
$T \propto \dfrac{1}{\sqrt{g'}}$ so, T increases

24. The length of a Second's pendulum is 1 m on the Earth. What will be the length of this pendulum on the Moon?

Sol. $T = 2\pi\sqrt{\dfrac{L}{g}}$

In both cases, T is same $\therefore \; L \propto g$

On the Moon, acceleration due to gravity is one-sixth of that on the Earth, so the required length = $\dfrac{1}{6}$m.

25. 

In the figure the block has a K.E. of 4 J and the spring has an elastic P.E. of 2 J when the block is at x = 1.5 cm. (a) What is the KE at x = 0? What are the elastic P.E. at (b) x = – 1.5 cm and (c) $x = -x_m$?

Sol. (a) At x = 0, the system is at its mean position, so, total energy is kinetic. Thus, KE at x = 0 is equal to total energy *i.e.*, (4 + 2) J or 6 J.

(b) It follows from the symmetry considerations that the elastic potential energy at x = –1.5 cm will be the same as elastic potential energy at x = +1 .5 cm, *i.e.*, 2 J.

(c) At $x = -x_m$, total energy is potential,

So, PE at $x = -x_m$ is (4 + 2) J = 6 J.

26. A linear harmonic oscillator of force constant 6×10^5 N/m and amplitude 4 cm, has a total energy 600 J. Calculate (1) Maximum K.E. (2) Minimum P.E.

Sol. (1) KE = $\dfrac{1}{2}kx^2 = \dfrac{1}{2}\times 6\times 10^5(4\times 10^{-2})^2 = 480\,\text{J}$

(2) Minimum PE = 600 – 480 = 120 J.

27. What would happen if sign of the force term in the equation $F = -kx$ is changed?

Sol. If the sign of the force term is changed. The force and hence acceleration will not be opposite to displacement. So, the body will not oscillate but it will be accelerated in the direction of displacement. So, the motion will become a linearly accelerated motion.

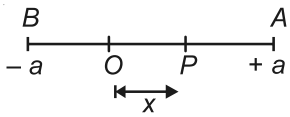
28. Distinguish between periodic motion and oscillatory motion.

Sol. **Periodic motion–**is that motion which is repeated identically after a fixed internal of time *e.g.*, revolution of Earth around the Sun is a periodic motion with a period of revolution of one year.

Oscillatory motion–is that motion in which a body moves to and fro about a fixed point (called mean or equilibrium position) in a definite internal of time *e.g.* motion of pendulum of a wall clock.

29. What do you understand by simple harmonic motion?

Sol. **Simple harmonic motion** is a special type of periodic motion in which a particle moves to and fro repeatedly about a mean (*i.e.*, equilibrium) position under action of a restoring force, which is always directed towards the mean position and whose magnitude at any instant is directly proportional to the displacement of the particle from the mean position at that instant.



Consider a particle executing SHM along x-axis between A and B with O as the mean position. $OA = +a$, $OB = -a$. Let at an instant t, the particle be at P, where $OP = x$, which is displacement of the particle from the mean position. The restoring force, $F = -kx$ where k is the force constant. Negative sign shows that the restoring force is always directed towards the mean position.

30. A vertical U tube of uniform cross-section contains water upto a height of 0.5 m. Water on one side is depressed a little and then released. Calculate time period and angular frequency of vibration. ($g = 10\ ms^{-2}$)

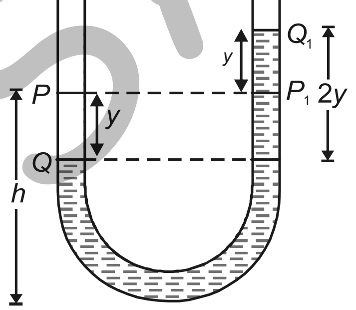
Sol. $T = 2\pi\sqrt{\dfrac{h}{g}} = 2\times 3.14\times\sqrt{\dfrac{0.5}{10}} = 6.28\times\sqrt{\dfrac{1}{20}} = 1.4\,s$

Angular frequency, $\omega = \dfrac{2\pi}{T} = \dfrac{2\times 22}{7\times 1.4} = 4.49\,s^{-1}$

Long Answer Type Questions :

31. Consider a liquid in a vertical U tube of uniform cross sectional area. Let the liquid in one limb be depressed a little and released causing the liquid column to oscillate. Show that the time period of oscillation does not depend upon the area of cross-section of the U tube and density of liquid but depends upon the length of liquid column and acceleration due to gravity.

Sol.



Consider a liquid of density ρ, contained in a vertical U tube of uniform area of cross-section A. In equilibrium position, let P and P_1 be the levels of liquid in the two limbs and L be the total length of liquid column from P to P_1.

Mass of the liquid in U tube, $m = LA\rho$ (ρ is density of liquid)

Let the liquid in left limb of U tube be depressed from P to Q by a small distance y *i.e.*, $PQ = y$ liquid level in right limb rises from P_1 to Q_1 by same distance y *i.e.*, P_1Q_1 = y. This is due to the small pressure difference maintained between the two columns.

The difference of levels of liquid in two limbs of U tube is $QQ_1 = 2y$

The weight of the liquid column of length $2y$ in U tube will provide a restoring force to mercury.

$\therefore$ Restoring force on liquid , F = – (weight of liquid column of height $2y$)

or $F = -(2yA)\rho g = -(2A\rho g)y$ ...(i)

From equation (i), we note that $F \propto y$ and F is directed towards equilibrium position. Hence, if pushing force is removed from the mercury column in U tube (*i.e.*, when the suction pump is removed) it will start executing linear SHM in U tube with equilibrium position as mean position.

In S.H.M. the restoring force $F = -ky$ ...(ii)

Comparing (i) and (ii), we have

Spring factor, $k = 2A\rho g$

Here, inertia factor, m = mass of liquid = $LA\rho$

Periodic time, $T = 2\pi\sqrt{\dfrac{\text{inertia factor}}{\text{spring factor}}}$

$$= 2\pi\sqrt{\frac{LA\rho}{2A\rho g}}$$

$$T = 2\pi\sqrt{\frac{L}{2g}} \qquad \text{...(iii)}$$

Frequency, $\nu = \dfrac{1}{T} = \dfrac{1}{2\pi}\sqrt{\dfrac{2g}{L}}$

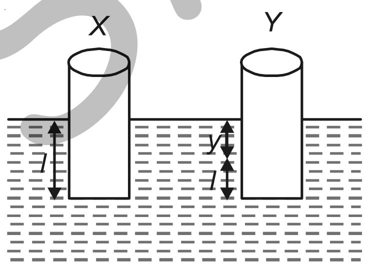
If h is the height of undisturbed mercury in each limb above the base of U tube, then $L = 2h$

$$\therefore \quad T = 2\pi\sqrt{\frac{2h}{2g}} = 2\pi\sqrt{\frac{h}{g}} \qquad \text{...(iv)}$$

From (iv), we find that time period does not depend upon the area of cross-section of the U tube, density of liquid but depends upon the length of the liquid column and acceleration due to gravity.

32. Consider a cylinder of mass m, length L, density of material ρ and uniform area of cross-section A. Find the expressions for time period and frequency of this floating cylinder as it is pushed a little and released.

Sol.



(Figure)

Consider a cylinder of mass m, length L, density of material ρ and uniform area of cross-section A. Therefore, $m = LA\rho$ ...(i)

Let the cylinder be floating vertically in a liquid of density σ. In equilibrium position, let l be the length of the cylinder dipping in the liquid shown by position X in figure. The upward thrust F_1 acting on the cylinder, according to Archimedes principle, will be equal to weight of the liquid displaced by length l of the cylinder.

So, $F_1 = (Al)\sigma g = Al\sigma g$

Weight of cylinder acting downward = mg

As, cylinder is in equilibrium position,

$mg = Al\sigma g$

or $m = Al\sigma$ ...(ii)

Let the cylinder be pushed down into the liquid through a small vertical distance y as shown by position Y in figure in this position, the total upward thrust F_2 acting on the cylinder is equal to the weight of the liquid displaced by the length $(\ell + y)$ of the cylinder.

$F_2 = A(l + y)\,\sigma\, g$ ...(iii)

Restoring force, F acting on the cylinder will be

$F = -(F_2 - mg)$.

$= -[A\,(l + y)\sigma g - A\ell\sigma g]$

$= -Ay\sigma g$

$= -(A\sigma g)y$ ...(iv)

Equation (iv) shown that the restoring force is directly proportional to displacement y and is directed towards the equilibrium position of cylinder. If the applied force is removed, the cylinder is left free, it will start executing linear SHM, about equilibrium position as mean position.

In SHM restoring force $F = -ky$ ...(v)

Comparing (iv) and (v),

Spring factor, $k = A\sigma g$

Inertia factor, $m = Al\rho$

$\therefore$ Periodic time, $T = 2\pi\sqrt{\dfrac{\text{Inertia factor}}{\text{Spring factor}}}$

$= 2\pi\sqrt{\dfrac{AL\rho}{A\sigma g}}$

$= 2\pi\sqrt{\dfrac{L\rho}{\sigma g}}$ ...(vi)

Using $m = Al\sigma$,

$$T = 2\pi\sqrt{\frac{Al\sigma}{A\sigma g}} = 2\pi\sqrt{\frac{l}{g}}$$

$\therefore$ Frequency, $\nu = \dfrac{1}{T} = \dfrac{1}{2\pi}\sqrt{\dfrac{g}{l}}$ ...(vii)

33. Derive an expression for the time period and frequency of a vertical spring.

Sol. Consider a light and highly elastic massless spring AB of spring constant k suspended from a rigid support at A, as shown in figure.

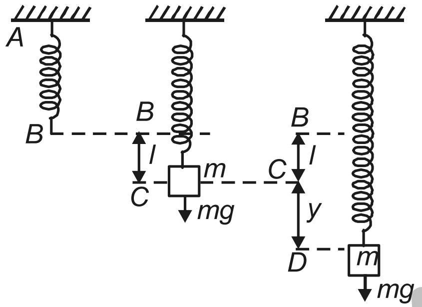


Figure (a) (b) (c)

The spring is unstretched and is in relaxed state. Let a body of mass m be attached to the lower end B of the spring. The spring gets stretched and suffers an extension, $BC = l$ figure (b). Let F_1 be the restoring force setup in the spring. Then

$$F_1 = -kl \qquad \text{...(i)}$$

Here, negative sign shows that the extension 'l' is directed downwards and restoring force F_1 is directed upwards. As the system is in equilibrium.

$$F_1 + mg = 0 \text{ or } F_1 = -mg \qquad \text{...(ii)}$$

Hence, $mg = kl$ or $k = \dfrac{mg}{l}$ ...(iii)

Let the body be pulled downwards through a small distance $CD = y(< l)$, figure (c). Now the total extension in the spring is $(l + y)$. If F_2 is the restoring force in this position, then

$$F_2 = -k\ (l + y) \qquad \text{...(iv)}$$

The effective restoring force will be $F = F_2 - F_1$

$$= -k\,(l + y) - (-lk)$$

$$= -ky \qquad \text{...(v)}$$

From equation (v), we note that $F \propto y$ and F is directed towards equilibrium position. Hence, if the pull from the suspended body is released, it will start executing SHM with C as mean position.

Here, spring factor = spring constant = k

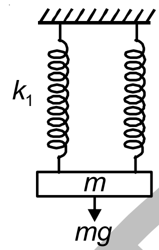
Inertia factor = mass of body = m

$\therefore$ Periodic time, $T = 2\pi\sqrt{\dfrac{\text{inertia factor}}{\text{spring factor}}} = 2\pi\sqrt{\dfrac{m}{k}} = 2\pi\sqrt{\dfrac{m}{mg/l}} = 2\pi\sqrt{\dfrac{l}{g}}$

Frequency, $\nu = \dfrac{1}{T} = \dfrac{1}{2\pi}\sqrt{\dfrac{g}{l}}$

34. Consider two springs of spring constants, k_1 and k_2. Let a body of mass m be suspended by the two springs in parallel combination. Derive an expression for time period and frequency of the combination. Also consider a special case $k_1 = k_2$ and get expression for time period.

Sol. A body of weight mg is suspended by two springs in parallel combination as shown in figure. Let the body be pulled downwards through a small distance y. If F_1 and F_2 are the restoring forces set up due to extension of springs, then



$F_1 = -k_1 y$

and $F_2 = -k_2 y$

Total restoring force, $F = F_1 + F_2 = -(k_1 + k_2)y$

If K is the spring constant of this combination then restoring force

$F = -Ky$

$\therefore \quad K = k_1 + k_2$

If the body is left free after pulling a little distance down, it will start executing S.H.M. of period T given by

$$T = 2\pi\sqrt{\frac{\text{Inertia factor}}{\text{Spring factor}}}$$

$$= 2\pi\sqrt{\frac{m}{K}}$$

$$= 2\pi\sqrt{\frac{m}{k_1 + k_2}} \quad ...(i)$$

$$\text{Frequency, } \nu = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{k_1 + k_2}{m}} \quad ...(ii)$$

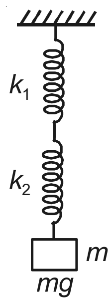
Special case, $k_1 = k_2 = k$, then $K = k + k = 2k$

$$T = 2\pi\sqrt{\frac{m}{2k}}$$

$$\text{and } \nu = \frac{1}{2\pi}\sqrt{\frac{2k}{m}}$$

35. Consider two springs of spring constant k_1 and k_2 connected in series combination. Let a body of mass m be suspended at the free end of the two springs in series combination. Derive an expression for time period and frequency. Also, consider a special case $k_1 = k_2$ and get an expression for time period.

Sol. A body of weight mg is suspended at the free end of the two springs in series combinations, as shown in figure. When the body is pulled downwards through a little distance y, the two spring suffer different extensions say, y_1 and y_2. But the restoring force is same in each spring.



$\therefore \quad F = -k_1 y_1$ and $F = -k_2 y_2$

or $y_1 = -\dfrac{F}{k_1}$ and $y_2 = -\dfrac{F}{k_2}$

$\therefore$ Total extension, $y = y_1 + y_2$

$$= \frac{-F}{k_1} - \frac{F}{k_2}$$

$$= -F\left(\frac{k_1 + k_2}{k_1 k_2}\right)$$

$$\text{or} \quad F = -\left(\frac{k_1 k_2}{k_1 + k_2}\right)y$$

If K is the spring constant of series combination, then, $F = -Ky$

$$\therefore \quad K = \frac{k_1 k_2}{k_1 + k_2}$$

If the body is left free after pulling down, it will execute SHM of period

$$T = 2\pi\sqrt{\frac{\text{Inertia factor}}{\text{Spring factor}}} = 2\pi\sqrt{\frac{m}{K}} = 2\pi\sqrt{\frac{m(k_1 + k_2)}{k_1 k_2}}$$

Frequency, $\nu = \dfrac{1}{T} = \dfrac{1}{2\pi}\sqrt{\dfrac{k_1 k_2}{m(k_1 + k_2)}}$

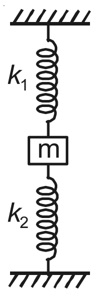
Special case : $k_1 = k_2 = k$

Then, $K = \dfrac{k \times k}{k + k} = \dfrac{k}{2}$

$$\therefore \quad T = 2\pi\sqrt{\frac{2m}{k}}$$

and $\nu = \dfrac{1}{2\pi}\sqrt{\dfrac{k}{2m}}$

36. Let a body of weight mg be connected in between two springs of spring constants k_1 and k_2 as shown in the figure and the body be pulled to one side through a small distance y. Derive an expression for time period and frequency. Also consider a special case $k_1 = k_2$ and get an expression for time period.



Sol. A body of weight mg is connected in between the springs. When it is pulled to one side through a small distance y, one spring gets compressed by length y and other spring gets stretched by length y. The restoring force F_1 and F_2 setup in both the springs will act in the same direction, then,

$F_1 = -k_1 y$ and $F_2 = -k_2 y$

Total restoring force, $F = F_1 + F_2$

$= -k_1 y - k_2 y$

$= -(k_1 + k_2)y$

If K is the spring constant of this combination of springs, then

$F = -Ky$

$\therefore \quad K = k_1 + k_2$

If the body is left free after pulling down a little, it executes SHM of period

$$T = 2\pi\sqrt{\frac{m}{k}}$$

$$= 2\pi\sqrt{\frac{m}{k_1 + k_2}}$$

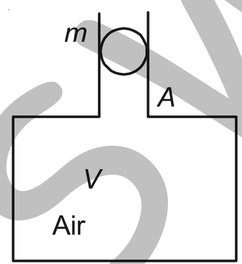
Frequency, $\nu = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{k_1 + k_2}{m}}$

Special case : $k_1 = k_2 = k$

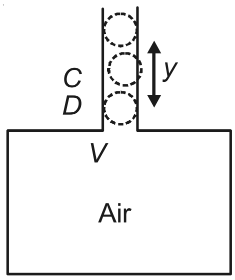
then $K = k + k = 2k$

and $T = 2\pi\sqrt{\frac{m}{2k}}$

37. Consider a ball in the neck of an air chamber. It just fits and can move up and down without any friction. Show that time period of oscillation depends upon bulk modulus of elasticity, mass of ball, area of cross section of neck and volume of air chamber. Assume pressure-volume variations of air to be isothermal.



Sol. Consider an air chamber of volume V with a long neck of uniform area of cross-section A, and a frictionless ball of mass m fitted smoothly in the neck at position C. The pressure of air below the ball inside the chamber = atmospheric pressure. Increase the pressure on the ball by a little amount p, so that the ball is depressed to position D where $CD = y$



There will be decrease in volume and hence, increase in pressure of air inside the chamber. The decrease in volume of air inside the chamber, $\Delta V = Ay$

Volumetric strain = $\frac{\text{Change in volume}}{\text{original volume}}$

$\therefore$ Bulk modulus of elasticity, $E = \frac{\text{stress (or increase in pressure)}}{\text{volumetric strain}}$

$$= \frac{-p}{Ay/V} = \frac{-pV}{Ay}$$

Here, –ve sign shows that increase in pressure will decrease the volume of air in the chamber.

$$p = \frac{-EAy}{V}$$

Due to this excess pressure, the restoring force acting on the ball is

$$F = pA = -\frac{EAy}{V}, A = \frac{-EA^2}{V}y \qquad \text{...(i)}$$

Since $F \propto y$ and –ve sign shows that the force is directed towards equilibrium position. If the applied increased pressure is removed from the ball, it will start executing linear SHM in the neck of chamber with C as the mean position.

In SHM, the restoring force, $F = -Ky$ ...(ii)

Comparing (i) and (ii),

$$\text{Spring factor, } K = \frac{EA^2}{V}$$

Inertia factor = mass of ball = m

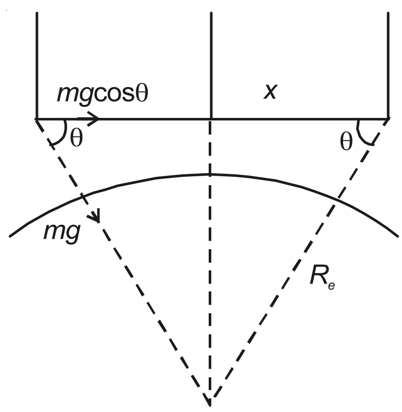
$$\text{Periodic time, } T = 2\pi\sqrt{\frac{\text{Inertia factor}}{\text{Spring factor}}}$$

$$= 2\pi\sqrt{\frac{m}{EA^2/V}}$$

$$= \frac{2\pi}{A}\sqrt{\frac{mV}{E}}$$

38. Derive an expression for the time period of a simple pendulum of infinite length.

Sol. The pendulum has infinite length, so, the pendulum bob would travel along the arc of a circle of infinite radius *i.e.*, along a straight line path as shown. The direction of the Earth's gravitational field is everywhere radially towards the centre of the Earth.



So, the direction of the gravitational field changes along the arc.

$$F = \frac{GM_em}{R_e^2} = mg$$

$$F_x = F\cos\theta = -F\frac{x}{R_e} = -\frac{GM_e m}{R_e^3}x$$

The –ve sign indicates that the force is directed opposite to the displacement.

or $F_x = -kx$ where $k = \frac{GM_e m}{R_e^3}$ is a constant.

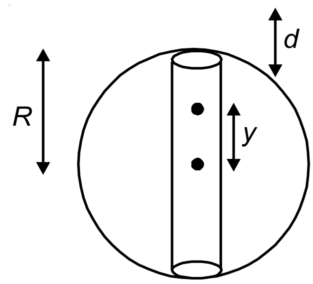
Time period of a simple harmonic oscillator

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{m}{GM_e m / R_e^3}}$$

$$= 2\pi\sqrt{\frac{R_e}{\frac{GM_e}{R_e^2}}} = 2\pi\sqrt{\frac{R_e}{g}}$$

39. A tunnel has been dug through the diameter of the Earth and a ball is released in it. Show that it executes SHM. Derive an expression for its time period. Also calculate this time period, given radius of Earth = 6.37×10^6 m and $g = 9.8$ ms^{-2}.

Sol. To establish that the motion of the ball is SHM,



$$g_d = -g\left(1 - \frac{d}{R}\right)$$

$$= -g\left(\frac{R-d}{R}\right)$$

or $$g_d = -\frac{g}{R}y \text{ where } y = R - d$$

$$T = 2\pi\sqrt{\frac{\text{displacement}}{\text{acceleration}}}$$

$$= 2\pi\sqrt{\frac{y}{gy/R}}$$

or $$T = 2\pi\sqrt{\frac{R}{g}}$$

$$T = 2 \times 3.14 \times \sqrt{\frac{6.37 \times 10^6}{9.8}}$$

$$= 5058\,\text{s} = 84.3 \text{ minute}$$

40. A simple pendulum is in a car which moves with (a) constant velocity (b) constant acceleration, 'a'. Discuss the effect on time period and the equilibrium position. Now a clock based on simple pendulum is taken to (i) Moon (ii) Centre of the Earth (iii) An artificial satellite (iv) A freely falling lift. Discuss the effect on time period, and also comment whether it will become slow or fast.

Sol. (a) When car is moving with constant velocity, there will be no change in time period and equilibrium position.

(b) When car is accelerating horizontally with acceleration 'a', effective acceleration due to gravity will be $\sqrt{g^2+a^2}$.

Time period will be $T = 2\pi\sqrt{\dfrac{L}{\sqrt{g^2+a^2}}}$

i.e., Time period T will decrease

Equilibrium position will not be vertical but will make an angle $\theta = \tan^{-1}\left(\dfrac{a}{g}\right)$ to the vertical in a direction opposite to the acceleration.

(i) When pendulum is taken to Moon, 'g' decreases and hence T will increase, so a clock based on simple pendulum will go slow.

In case of (ii), (iii) and (iv) the value of $g = 0$

So, T will become infinity. That means the pendulum will not oscillate and will remain where it is left.

41. The bob of a simple pendulum has charge Q and is oscillating in a uniform electric field. Discuss the effect on time period when uniform electric field is (i) In the direction of g (ii) Opposite to direction of g (iii) Perpendicular to direction of g.

Sol. If the bob of simple pendulum has charge Q and is oscillating in a uniform electric field which is (i) in the direction of g

Electric force QE will support the weight mg and the effective acceleration due to gravity, $g' = g + \dfrac{QE}{m}$

So, time period $T' = 2\pi\sqrt{\dfrac{L}{g+\dfrac{QE}{m}}}$ is less than the normal value.

(ii) If the electric force is opposite to weight effective acceleration due to gravity, $g' = g - \dfrac{QE}{m}$.

So, time period $T' = 2\pi\sqrt{\dfrac{L}{g-\dfrac{QE}{m}}}$, which is more than the normal value.

(iii) If the electric force is perpendicular to the weight, then effective acceleration due to gravity, $g' = \sqrt{g^2+(QE/m)^2}$ and time period

$T' = 2\pi\sqrt{\dfrac{L}{\sqrt{g^2+(QE/m)^2}}}$ and it will be less than the normal value.

42. One end of a V-tube containing mercury is connected to a suction pump and the other end to atmosphere. The two arms of the tube are inclined to horizontal at an angle of 45º each. A small pressure difference is created between two columns when the suction pump is removed. Will the column of mercury in V-tube execute SHM? Neglect capillary and viscous forces. Find the time period of oscillation.

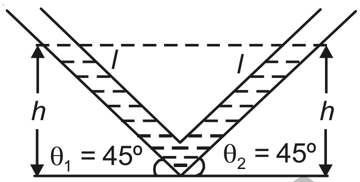
Sol. Let the mercury be depressed by x in left arm of v-tube so it rises by x along the length of the tube in the right arm of V tube. The restoring force to the mercury is provided by the pressure difference in the two arms.

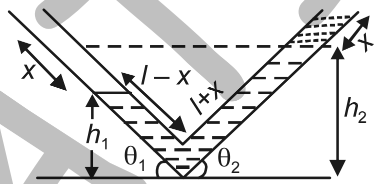
$\therefore \quad F = -(\Delta P)A$ ...(i)

Here $P_1 = h_1\rho g \sin\theta_1 = (l - x)\sin\theta_1 \times \rho g \sin\theta_1$

$= (l - x)\rho g \sin^2\theta_1$

$= \dfrac{(l - x)\rho g}{2} \quad (\because \theta_1 = 45^\circ)$





and $P_2 = (l + x)\,\rho g \sin^2\theta_2$

$= \dfrac{(l + x)\rho g}{2}$

$\therefore \quad \Delta P = (P_2 - P_1)$

$= \dfrac{\rho g}{2}(l + x - l + x)$

$= \rho g x$

Hence, restoring force, $F = -\rho g A x$

$\therefore$ Acceleration of mercury column $= a = \dfrac{F}{m} = -\dfrac{\rho g A x}{m}$

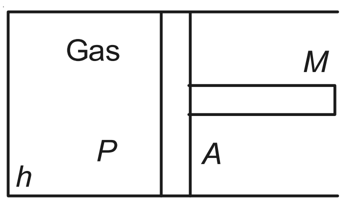
But, $m = lA\rho$

$\therefore \quad a = \dfrac{-\rho g A x}{lA\rho} = -\left(\dfrac{g}{l}\right)x$

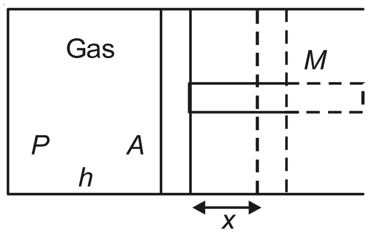
Hence, motion of mercury column is S.H.M.

$\therefore \quad T = 2\pi\sqrt{\dfrac{x}{a}} = 2\pi\sqrt{\dfrac{l}{g}}$

43. A cylindrical piston of mass m slides smoothly inside a long cylinder closed at one end, enclosing a certain mass of gas. The cylinder is kept with its axis horizontal. Show that piston oscillates simple harmonically (when it is disturbed from its equilibrium position) with a period of $2\pi\sqrt{\dfrac{Mh}{PA}}$



Sol.



Let the piston be displaced through distance x towards left, then volume decreases, pressure increases. If ΔP is increase in pressure and ΔV is decrease in volume, then considering the process to take place gradually (*i.e.*, isothermal)

$$P_1V_1 = P_2V_2$$

$$\Rightarrow \quad PV = (P + \Delta P)(V - \Delta V)$$

$$PV = PV + \Delta PV - P\Delta V - \Delta P\Delta V$$

$$\Delta PV - P\Delta V = 0 \text{ (neglecting } \Delta P\Delta V)$$

$$\Delta P(Ah) = P(Ax)$$

$$\Rightarrow \quad \Delta P = \frac{Px}{h}$$

This excess pressure is responsible for providing the restoring force (F) to the piston of mass m,

Hence, $F = \Delta PA = \dfrac{PAx}{h}$

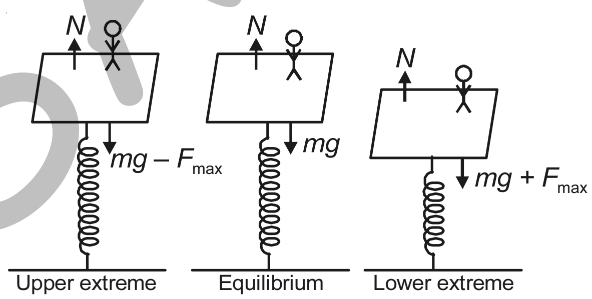
Comparing with $|F| = Kx$

$$k = M\omega^2 = \frac{PA}{h}$$

$$\omega = \sqrt{\frac{PA}{h}} \Rightarrow T = \sqrt{\frac{Mh}{PA}}$$

44. A boy weighing 30 kg stands on the horizontal platform of a spring balance. Show that the reading of the weight recorded by spring balance fluctuates between 20 kgf and 40 *kgf* when the platform starts executing SHM of amplitude 0.1 m and frequency $\dfrac{2}{\pi}$ cps.

Sol.



The maximum force acting on the body executing SHM is $m\omega^2 a = m(2\pi\nu)^2 a = 30 \times \left(2\pi\dfrac{2}{\pi}\right)^2 \times 0.1\,\text{N}$

$\approx$ 5 kgf and this force is towards the mean position.

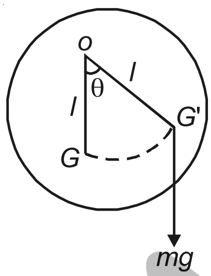
The reaction of the force on the platform is away from the mean position. It reduces the weight of boy on upper extreme *i.e*, net weight = 30 – 10 = 20 kgf

This force adds to weight at lower extreme position, *i.e.*, net weight becomes = 30 + 10 = 40 kgf

$\therefore$ The reading of the weight recorded by spring balance fluctuates between 20 kgf and 40 kgf.

45. A rigid body suspended from a fixed support, constitutes a physical pendulum if it is capable of swinging in a vertical plane about some axis passing through the fixed support. Derive expressions for its time period and frequency. Mention the factor for which the motion is angular simple harmonic.

Sol. Let a rigid body be suspended from a fixed support. If constitutes a physical pendulum if it is capable of swinging in a vertical plane about some axis passing through the fixed support.



Let l = the separation between the point of suspension (O) and the centre of mass (G)

θ = the angle through which it is displaced from the equilibrium position.

m = mass of the body

I = the moment of inertia of the body about the axis of rotation.

Torque, $\tau = -\,mgl(\sin\theta)$

$= (-\,mgl)\theta$ (for small θ)

Angular acceleration, $\alpha = \dfrac{\tau}{I} = \dfrac{mgl}{I}\theta = -\omega^2\theta$

where $\omega^2 = \dfrac{mgl}{I}$

$\therefore$ Time period, $T = \dfrac{2\pi}{\omega} = 2\pi\sqrt{\dfrac{I}{mgl}}$

QUESTION ALIKE
(Waves)

Very Short Answer Type Questions :

1. A wave travelling along a string is described by, $y(x, t) = 0.05 \cos(250x - 30t)$, where x and y are in metre and t in s. What is the amplitude of the wave?

Sol. 5 cm

2. How does the velocity of sound relate to the density of the medium?

Sol. $v \propto \dfrac{1}{\sqrt{\rho}}$

3. What is the distance between a compression and its nearest rarefaction in a longitudinal wave?

Sol. $\lambda/2$

4. A person is moving away from the source, with a speed equal to the velocity of sound propagation. Will the person hear any sound produced by the source?

Sol. No, the person will not hear any sound,

$\because \quad \nu' = \left(\dfrac{v - v}{v}\right)\nu$ = zero

5. If the distance between a source of sound and the listener is doubled, what is the change in frequency observed by the listener?

Sol. No change. Frequency is independent of the distance between source and listener. It depends on their relative motion rather.

6. Two harmonic waves of frequencies 50 Hz and 54 Hz are superposed. What is the beat frequency observed?

Sol. Beat frequency $m = \nu_2 - \nu_1 = 4$ Hz

7. A travelling wave is incident on a boundary and gets reflected. If the equations of the incident and the reflected waves are $y_i(x, t) = a \sin(kx - \omega t)$ and $y_r(x, t) = -a \sin(kx + \omega t)$ respectively, then check whether the boundary is rigid or open?

Sol. Reflected wave is 180° out of phase from the incident wave. Therefore, the boundary must be a rigid boundary.

8. In a certain wave propagation, the constituents of the medium are vibrating along the direction of propagation of the wave. Is this wave transverse or longitudinal?

Sol. Longitudinal wave.

9. What is the relation between angular frequency (ω) and propagation constant (k) of a harmonic wave?

Sol. $\frac{\omega}{k} = v$; where v is the velocity of the wave.

10. Which kind of waves are propagated through solids, *i.e*. longitudinal or transverse?

Sol. Both longitudinal as well as transverse waves.

Short Answer Type Questions :

11. Explain why the propagation of longitudinal waves can be studied on a helical spring but not on a string.

Sol. In a longitudinal wave, the constituents of the medium oscillates along the wave motion. String cannot be stretched beyond its natural length significantly. Therefore, visible compressions and rarefactions are not possible in a string. That is why longitudinal waves cannot be studied on a string. However they can be easily produced in a helical spring.

12. The range of audible frequency is 20 Hz to 20000 Hz. Express this range in terms of

(i) Time period

(ii) Wavelength

for a medium where the velocity of sound is 330 m/s.

Sol. (i) $\nu = \frac{1}{T}$

$\therefore$ 20 Hz frequency is equivalent to $\frac{1}{20}$ s *i.e.*, 0.05 s and 20,000 Hz equals $\frac{1}{20000}$ s or 0.5×10^{-4} s.

$\therefore$ Audible range in terms of time period is 0.5×10^{-4} s to 0.05 s.

(ii) $\lambda = \frac{v}{\nu}$

$\therefore \quad \lambda_1 = \frac{330 \text{ m/s}}{20 \text{ Hz}} = 16.5 \text{ m}$

and $\lambda_2 = \frac{330}{20{,}000} = 16.5 \times 10^{-3} \text{ m}$

$\therefore$ Audible range in terms of wavelength is 16.5 m to 16.5×10^{-3} m for the given medium.

13. Calculate the number of compressions and rarefactions which will pass an observer in 1 hr, if the velocity of sound wave is 332 m/s and the wavelength is 2 m.

Sol. One compression and one rarefaction together constitude one wave cycle.

Number of cycles passing the observer every second,

Frequency = $\frac{332 \text{ m/s}}{2 \text{ m}}$ = 166 Hz

$\therefore$ No. of cycles passing him in one hour = $166 \times 60 \times 60 = 597600$

14. The equation given below represents a stationary wave set up in a medium

$$y = 5\cos\left(\frac{2\pi x}{3}\right)\sin(20\pi t)$$

where x and y are in cm and t is in s.

Calculate the amplitude, wavelength and velocity of the component waves.

Sol. Compare the equation with

$$y = 2a \cos kx \sin\omega t$$

We get, $a = \frac{5}{2}$cm, $k = \frac{2\pi}{3}$rad cm^{-1} and $\omega = 20\pi$ s^{-1}

Now, $k = \frac{2\pi}{\lambda}$

$$\Rightarrow \quad \lambda = \frac{2\pi}{k} = \frac{2\pi}{2\pi/3} = 3 \text{ cm}$$

and velocity $v = \frac{\omega}{k} = \frac{20\pi}{2\pi/3} = 30$ cm/s

15. The pressure and volume variations in the gases, when sound propagates through them are adiabatic and not isothermal. Explain.

Sol. The pressure and volume variations in the gases when sound propagates through them are so fast that heat cannot flow to the surroundings or from the surroundings to keep the temperature constant. Therefore, the variations are adiabatic and not isothermal.

16. A tuning fork produces sounds of wavelengths 2 m and 3.8 m while vibrating in air and hydrogen respectively. Calculate the velocity of sound in hydrogen if the velocity of sound in air is 332 m/s.

Sol. $\frac{v_2}{v_1} = \frac{\lambda_2}{\lambda_1}$

v_2 = 630.8 m/s

17. Calculate the temperature at which the velocity of sound in oxygen will be the same as that in hydrogen at 50°C.

Sol. $v = \sqrt{\frac{\gamma RT}{M}}$

$\because$ γ is same for both O_2 and H_2, $\therefore$ for $v_{H_2} = v_{O_2}$

$$\Rightarrow \quad \frac{T_{H_2}}{M_{H_2}} = \frac{T_{O_2}}{M_{O_2}}$$

$\Rightarrow$ T_{O_2} = 5168 K or 4895°C

18. A stone is dropped into a well and the impact of sound is heard 5.4 s later. If the depth of the well is 122.5 metre then calculate the velocity of sound.

Sol. Let time taken by the stone to reach the water surface is t_1, then

Depth of the well, $s = ut_1 + \frac{1}{2}at_1^2$

$$122.5 = (0 \times t_1) + \frac{1}{2} \times 9.8\, t_1^2$$

$$\Rightarrow \quad t_1^2 = \frac{122.5}{4.9} = 25$$

$t_1 = 5$ s.

$\therefore$ Time taken by the sound to reach the listener = 5.4 – 5 = 0.4 s.

$\therefore$ Velocity of sound = $\frac{122.5}{0.4}$ = 306.25 m/s

19. Explain why the pitch of a car horn we hear is decreased as the car moves away from us.

Sol. When the car is going away from the listener, the pitch or frequency of the sound decreases. This is due to Doppler effect. The observed frequency heard by the listener is given by

$$\nu' = \frac{v}{(v + v_s)}\nu$$

$\Rightarrow$ $\nu' < \nu$

$\therefore$ Observed pitch is less than the actual pitch.

20. A sound wave of frequency 50 Hz is propagating in space with speed 350 m/s. Calculate the phase difference of the wave at two points separated by 3.5 m.

Sol. Wavelength(λ) = $\frac{v}{f} = \frac{350 \text{ m/s}}{50 \text{ Hz}} = 7$ m

Phase of a wave = $kx - \omega t$

At a given instant, phase difference between two points x_1 and x_2,

$$= \frac{2\pi}{\lambda}(x_2 - x_1) = \frac{2\pi \times (3.5)}{7} = \pi \text{ radian}$$

21. Calculate the distance travelled by the sound wave produced by a tuning fork of frequency 120 Hz after completing 60 vibrations. Given that the speed of sound in air is 332 m/s.

Sol. Wavelength of sound(λ) = $\frac{v}{f}$

$$\Rightarrow \lambda = \frac{332}{120} = 2.76 \text{ m}$$

By the time the wave completes n vibrations, it travels a distance $n\lambda$.

$\therefore$ Distance travelled by the sound = $n\lambda$

= 60×2.76

= 165.6 m

22. If the first overtone of a closed pipe of length 23 cm has the same frequency as the 2nd overtone of an open pipe, then calculate the length of the open pipe.

Sol. 1st overtone of closed pipe is $\nu_1 = \frac{3v}{4L}$

2nd overtone of open pipe is $\nu_2' = \frac{3v}{2L'}$

Now, $\nu_1 = \nu_2'$

$\Rightarrow \quad \frac{3v}{4L} = \frac{3v}{2L'} \Rightarrow L' = 2L$

$= 46$ cm

23. A tuning fork produces 7 beats per second when sounded together with a tuning fork of frequency 220 Hz. When the first tuning fork is loaded with a little wax then the number of beats produced becomes 3. What is the frequency of the first tuning fork?

Sol. Beat frequency $m = (\nu_1 - \nu_2)$ or $(\nu_2 - \nu_1)$

$\Rightarrow \quad \nu_1 = 227$ Hz or 213 Hz

On loading the first fork, the number of beats produced per second decreases therefore, $\nu_1 = 227$ Hz.

24. The displacements at a point due to two waves are given by $y_1 = 2.7 \sin(32\pi t)$ and $y_2 = 3\sin(52\pi t)$. Calculate the number of beats produced per second.

Sol. Frequency of the 1st wave $\nu_1 = 16$

Frequency of the 2nd wave $\nu_2 = 26$

$\therefore \quad$ Beat frequency $(m) = \nu_2 - \nu_1 = 26 - 16 = 10$ Hz

25. If an open pipe of length 34 cm resonates with a frequency of 2500 Hz then which of the harmonics does, it represent. Given that velocity of sound in air is 340 m/s.

Sol. Frequency of normal modes in open pipe = $\frac{nv}{2L}$

$$2500 = \frac{n \times 340}{2 \times 0.34}$$

$$n = \frac{2500 \times 2}{340} \times \frac{34}{100}$$

$= 5 \qquad \therefore$ Fifth harmonic

26. Two engines pass each other in opposite directions each with a speed of 50 m/s relative to air. One of them is emitting a note of frequency 200 Hz. Calculate the frequency heard by a man in the other engine after they have passed each other. Given that the velocity of sound is 350 m/s.

Sol. $\nu' = \left(\frac{v - v_L}{v + v_s}\right)\nu$

$$= \left(\frac{350 - 50}{350 + 50}\right) \times 200$$

$$= \frac{300}{400} \times 200 = 150 \text{ Hz}$$

27. Calculate the fundamental frequency of a metre long stretched string, fixed at both ends with mass 0.10 kg under a tension of 1600 newton.

Sol. Velocity of wave in the string is

$$v = \sqrt{\frac{T}{\mu}}$$

$$= \sqrt{\frac{1600 \times 1}{0.10}}$$

$$= 400 \text{ m/s}$$

Fundamental frequency $= \frac{v}{2L}$

$$= \frac{400}{2 \times 1}$$

$$= 200 \text{ Hz}$$

28. A 1.5 m long rope is stretched between two rigid supports with a tension that makes the speed of transverse waves 48 m/s. What are the frequency of (i) the fundamental mode (ii) the second overtone?

Sol. Fundamental frequency $\nu_1 = \frac{v}{2L} = \frac{48}{2 \times 1.5} = 16 \text{ Hz}$

2nd overtone frequency $\nu_3 = 3\nu_1 = 48$ Hz

29. A metal bar with a length of 3 m has density 3200 kg/m^3. Longitudinal sound waves take 2.8×10^{-4} s to travel from one end of the bar to the other. What is Young's modulus for this bar?

Sol. $v = \sqrt{\frac{Y}{\rho}}$

$$\Rightarrow \quad Y = v^2\rho = \left(\frac{L}{t}\right)^2 \rho$$

$$= \left(\frac{3}{2.8 \times 10^{-4}}\right)^2 \times 3200$$

$$= (1.07 \times 10^4)^2 \times 3200$$

$$= 36.73 \times 10^{10} \text{ Pa}$$

30. A car alarm is emitting sound waves of frequency 500 Hz. You are on a motorcycle, travelling directly away from the car. How fast must you be travelling if you detect a frequency 470 Hz? Given the velocity of sound in air is 320 m/s.

Sol. $\nu' = \left(\frac{v - v_L}{v}\right)\nu$

$$\Rightarrow \quad 470 = \left(\frac{320 - v_L}{320}\right) 500 \Rightarrow v_L = 19.2 \text{ m/s}$$

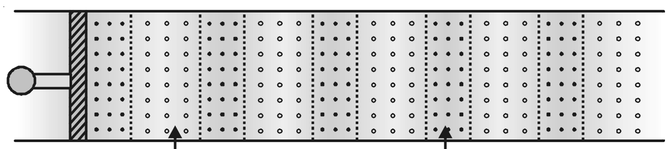
Long Answer Type Questions :

31. What are the longitudinal waves? How longitudinal waves can be propagated in a long air filled pipe with a piston at one end?

Sol. Longitudinal waves : These are the waves in which the constituents of the medium oscillate along the direction of wave propagation.

Propagation of longitudinal waves in a long air-filled pipe

Consider a long air-filled pipe with a piston at one end. If we suddenly move the piston rightward and then leftward, we can generate a pulse of compression and rarefaction along the pipe.



If we push in and pull out the piston continuously and periodically, a sinusoidal disturbance is generated along the pipe. Because the motion of the elements of air is parallel to the direction of the wave's propagation, the motion is said to be longitudinal and the wave is said to be a longitudinal wave.

32. Give some important characteristics of wave motion.

Sol. Characteristics of wave motion

1. A wave is a kind of disturbance that travels through a medium, acompanied by a transfer of energy.
2. In a wave propagation, constituents of medium oscillate about their mean positions.
3. There is no net transfer of medium when wave propagates through it.
4. Velocity of wave propagation is different for different media.
5. A material medium is required for the propagation of mechanical wave.
6. Electromagnetic waves do not require any material medium for their propagation
7. Velocity of constituent particles of the medium is maximum at their mean position and minimum at their extreme position.

33. Discuss Laplace correction in Newton's formula. Calculate the speed of sound in air using Newton's corrected formula, given that the density of air is 1.293 kg/m^3 and γ_{air} = 1.40.

Sol. Laplace's correction in Newton's formula

Newton assumed that the pressure variations in a medium during propagation of sound are isothermal. Laplace pointed out that the pressure variations in the gases when sound propagates are so fast that heat could not flow to surroundings or from surroundings to keep the temperature constant. Therefore, the variations are adiabatic and not isothermal.

For an adiabatic process,

PV^γ = constant

$P\Delta(V^\gamma) + V^\gamma \Delta P = 0$

$P\gamma V^{\gamma-1}\Delta V + V^\gamma \Delta P = 0$

$$\frac{-\Delta P}{\Delta V / V} = \gamma P$$

Further, the adiabatic Bulk Modulus is given by

$$B_{ad} = \frac{-\Delta P}{\Delta V / V}$$

This implies that $B_{ad} = \gamma P$

Therefore, the speed of sound in gas is

$$v = \sqrt{\frac{\gamma P}{\rho}}$$

Calculation of velocity of sound

$$v = \sqrt{\frac{\gamma P}{\rho}}$$

Density of air = 1.293 kg/m^3 and γ_{air} = 1.40

Further atmospheric pressure $P = 1.01 \times 10^5$ Pa

$$\Rightarrow \quad v = \sqrt{\frac{1.40 \times 1.01 \times 10^5}{1.293}} = 331.3 \text{ m/s}$$

34. State and explain the principle of superposition of waves. Hence, give conditions for (i) maximum amplitude (ii) minimum amplitude.

Sol. According to the principle of superposition of waves, each wave pulse moves as if others are not present. The constituents of the medium therefore suffer displacement due to both and since displacements can be positive and negative, the net displacement is an algebraic sum of the two superposing waves. This discussion can be extended to any number of waves.

Let us consider two sinusoidal waves of same wavelength and frequency travelling in same direction along a stretched string. Further let the waves have same amplitude and a constant phase difference of ϕ.

Accordingly the two waves are described by the functions.

$$y_1(x, t) = a \sin(kx - \omega t)$$

$$y_2(x, t) = a \sin(kx - \omega t + \phi)$$

From the principle of superposition, the resultant wave is

$$y(x, t) = y_1(x, t) + y_2(x, t)$$

$$= a \sin(kx - \omega t) + a \sin(kx - \omega t + \phi)$$

$$= \left[2a \cos\frac{\phi}{2}\right] \sin\left(kx - \omega t + \frac{\phi}{2}\right)$$

(i) **For maximum amplitude**

$\phi = 0$ or $2n\pi$ where $n = 0, 1, 2……$

$y(x, t) = 2a \sin(kx - \omega t)$

i.e., the resultant wave has a maximum amplitude of $2a$.

(ii) **For minimum amplitude**

$\phi = (2n - 1)\pi$ where n = 0, 1, 2……

$\Rightarrow$ $y(x, t) = 0$

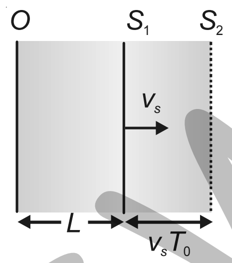
i.e., the resultant wave has a minimum amplitude of zero.

35. Explain Doppler effect in sound. Obtain an expression for the apparent frequency of sound, when the source is moving and the observer is at rest.

Sol. Doppler Effect

Whenever there is a relative motion between the source of sound and an observer, the frequency of the sound received or heard by the observer is different from the frequency of sound produced by the source. This is called the Doppler effect.

Apparent frequency of sound when **source is moving** and **observer is at rest :**



Let us consider a source of sound producing a wave of angular frequency ω, velocity v and time period T_0 and moving with velocity v_s. Let initially at $t = 0$, the source is at S_1 and observer is at O, where $OS_1 = L$. At this instant, the source produces a crest, which reaches the observer at time $t = t_1$, which is given by

$$t_1 = \frac{L}{v}$$

Now At time $t = T_0$, the source is at S_2, where the distance $S_1S_2 = v_sT_0$.

At this time, a second crest is produced by the source which reaches the observer at $t = t_2$

Where $$t_2 = T_0 + \frac{(L + v_sT_0)}{v}$$

Similarly, 3rd crest reaches the observer at $t = t_3$

$$t_3 = 2T_0 + \frac{(L + 2v_sT_0)}{v}$$

The source emits its $(n + 1)^{th}$ crest and this reaches the observer at time

$$t_{n+1} = nT_0 + \frac{(L + nv_sT_0)}{v}$$

Therefore, n crests are counted by the observer in time interval $= t_{n+1} - t_1$

$$= nT_0 + \frac{(L + nv_sT_0)}{v} - \frac{L}{v}$$

$$= nT_0 + \frac{nv_sT_0}{v}$$

$\therefore$ Time period of the wave observed by the observer is

$$T = \left[nT_0 + \frac{nv_sT_0}{v}\right] / n$$

$$T = T_0\left(1 + \frac{v_s}{v}\right)$$

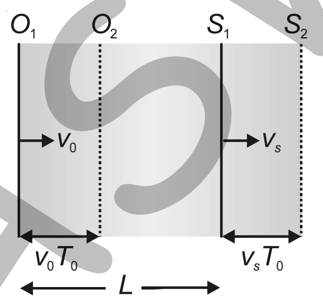
Also the observed frequency is given by

$$\nu = \nu_0\left(1 + \frac{v_s}{v}\right)^{-1}$$

$$\boxed{\nu = \nu_0\left(\frac{v}{v + v_s}\right)}$$

36. Using Doppler effect in sound, obtain an expression for the apparent frequency of sound, when source and listener both are moving.

Sol. Let us consider that both the source and observer are moving with velocity v_s and v_0 respectively as shown in the figure.



The source emits a wave of angular frequency (ω), velocity v and time period T_0.

Initially, at time $t = 0$ the observer and source are at O_1 and S_1 respectively where $O_1S_1 = L$.

At this instant the source emits a crest.

Since the observer is moving with velocity v_0 the relative velocity of the wave w.r.t. the observer is $(v + v_0)$.

Time taken by the first crest to reach the observer is $t_1 = \dfrac{L}{(v + v_0)}$

At time $t = T_0$, the observer is at O_2 and the source is at S_2 and the source produces a second crest, which reaches the observer at time $t = t_2$

$$t_2 = T_0 + \frac{L + (v_s - v_0)T_0}{(v + v_0)}$$

Where the distance $O_2S_2 = L + (v_s - v_0)T_0$

Similarly, 3rd crest reaches the observer at $t = t_3$, Where $t_3 = 2T_0 + \dfrac{L + 2(v_s - v_0)T_0}{(v + v_0)}$

Proceeding in the same manner the source emits its $(n + 1)^{th}$ crest and this reaches the observer at time

$$t_{n+1} = nT_0 + \left[\frac{L + n(v_s - v_0)T_0}{(v + v_0)}\right]$$

Therefore, n crests are counted by the observer in the time interval $= t_{n+1} - t_1$

$$= nT_0 + \left[\frac{L + n(v_s - v_0)T_0}{v + v_0}\right] - \frac{L}{(v + v_0)}$$

$\therefore$ Time period of the wave as observed by the observer is

$$T = \left[nT_0 + \frac{n(v_s - v_0)T_0}{(v + v_0)}\right] / n$$

$$T = T_0\left(1 + \frac{v_s - v_0}{(v + v_0)}\right)$$

$$T = T_0\left(\frac{v + v_s}{v + v_0}\right)$$

Also the observed frequency is given by

$$\boxed{\nu = \nu_0\left(\frac{v + v_0}{v + v_s}\right)}$$

37. A travelling wave equation is given by $y = 4 \sin\left(8\pi t + \dfrac{\pi x}{12}\right)$, where x and y are in metre and t is in second.

(i) Calculate amplitude, frequency, angular frequency, time period and initial phase.

(ii) Calculate the number of compressions and rarefactions which will pass an observer in 4 minute.

Sol. (i) Compare the given equation with standard equation of travelling wave.

$$y = a \sin(\omega t + kx)$$

We get $a = 4$ m, $\omega = 8\pi$ rad s^{-1}; $k = \dfrac{\pi}{12}$ rad m^{-1}

Therefore, amplitude = 4 m

Angular frequency = 8π rad s^{-1}

Frequency $(\nu) = \dfrac{\omega}{2\pi} = 4$ Hz

Time period (T) = $\frac{1}{\nu} = \frac{1}{4}$ s = 0.25 s

Initial phase (ϕ_0) = 0

(ii) Wave velocity $(v) = \frac{\omega}{k} = \frac{8\pi \times 12}{\pi} = 96$ m/s

Distance travelled by wave in 4 min = $96 \times 4 \times 60$

= 23040 m

Wavelength of the wave (λ) = $\frac{2\pi}{k} = \frac{2\pi}{\pi} \times 12 = 24$ m

$\therefore$ Number of compressions and rarefactions which passes an observer in 4 min = $\frac{23040}{24} = 960$

38. (i) At what temperature will the velocity of sound in air be double that at 0°C?

(ii) Calculate the temperature at which the velocity of sound in oxygen will be equal to that in nitrogen at 7°C.

Sol. (i) Velocity of sound $v = \sqrt{\frac{\gamma RT}{M}}$

$\Rightarrow \quad v \propto \sqrt{T}$

$\Rightarrow \quad \frac{v_T}{v_0} = \sqrt{\frac{T_2}{T_0}} = \sqrt{\frac{273+t}{273}}$

$\Rightarrow \quad 2 = \sqrt{\frac{273+t}{273}} \Rightarrow t = 819^\circ$ C

(ii) Molecular wt. of oxygen = 32

Molecular wt. of nitrogen = 28

Now velocity of sound $\nu = \sqrt{\frac{\gamma RT}{M}}$

$\Rightarrow \quad \frac{v_O}{v_N} = \sqrt{\frac{T_O}{M_O} \times \frac{M_N}{T_N}}$

$1 = \sqrt{\frac{(273+t) \times 28}{32 \times (280)}}$

t = 47°C

39. A man adjusts his watch by the sound of a signal from a distant tower. He observes that his watch is slow by 3 s. Calculate the distance of the tower from the man, if the temperature of air is 25°C and velocity of sound is 332 m/s at 0°C.

Sol. Velocity of sound at 0°C is 332 m/s

Velocity of sound at 25°C is v_T, then

$$\frac{v_T}{v} = \sqrt{\frac{273+t}{273}}$$

$$v_T = 332\sqrt{\frac{298}{273}}$$

$$= 332 \times 1.04478 = 346.87 \text{ m/s}$$

Distance of the tower from the man = $v_T \times$ (the delay shown by his watch)

$$= 346.87 \times 3 = 1040.60 \text{ m}$$

40. A tuning fork of frequency 250 Hz produces 4 beats per s. when sounded with another tuning fork. On loading the second tuning fork with a little wax, if the number of beats produced per second

(i) Decreases then what is the frequency of the second fork?

(ii) Increases then what is the frequency of the second fork?

Sol. Number of beats per second (m) = $(\nu_1 - \nu_2)$ or $(\nu_2 - \nu_1)$

$\Rightarrow$ $\nu_2 = (\nu_1 + m)$ or $(\nu_1 - m)$

Given that ν_1 = 250 Hz

$\Rightarrow$ Frequency of 2nd tuning fork is either 254 or 246 Hz.

(i) In 1st case, since on loading the frequency of tuning fork decreases, hence its frequency cannot be 246 because in that case the number of beats produced per second will increase. Therefore, frequency of the second tuning fork is 254 Hz.

(ii) In this case, since on loading the frequency of tuning fork decreases, hence its frequency cannot be 254 Hz because in that case the number of beats produced per second will decrease. Therefore, frequency of the second tuning fork is 246 Hz.

41. (i) A tuning fork of frequency 216 cps produces 3 beats per s, when sounded with a second fork. On filing the prongs of the second tuning fork, the number of the beats produced per second becomes 5. What is its actual frequency?

(ii) What will be the frequency of the second fork if number of beats produced per second becomes 2?

(iii) What will be the frequency of the second fork if not a single beat is heard?

Sol. No. of beats per second $m = (\nu_1 - \nu_2)$ or $(\nu_2 - \nu_1) = 3$

$\Rightarrow$ $\nu_2 = (\nu_1 + m)$ or $(\nu_1 - m)$

Given that ν_1 = 216 Hz

$\Rightarrow$ Frequency of 2nd tuning fork is either 219 Hz or 213 Hz.

(i) On filing the prongs of the 2nd fork, the frequency of the sound produced by it'll increase. Now, the number of beats produced becomes 5. Here no. of beats produced per second. increases, therefore ν_2 = 219 Hz.

(ii) In this case, on filing the prongs, no. of beats produced becomes 2. Hence, its frequency cannot be 219 Hz. Because here no. of beats produced per second decreases, therefore ν_2 = 213 Hz.

(iii) In this case, on filing the prongs, no beat is heard at all. Hence, again no. of beats produced per second decreases, therefore ν_2 = 213 Hz.

42. The equation given below represents a stationary wave set up in a medium $y = 3\cos(3\pi x)\sin(30\pi t)$, where x and y are in cm and t is in second.

(i) Calculate the amplitude, wavelength and velocity of the component waves.

(ii) Calculate the distance between two adjacent nodes.

(iii) Calculate the number of beats produced.

Sol. (i) Comparing the given equation with stationary wave equation

$$y = 2a\cos(kx)\sin(\omega t)$$

We have, $a = \frac{3}{2}\text{cm}$, $k = 3\pi$ rad cm^{-1}

and $\omega = 30\pi$ rad s^{-1}

Amplitude of the component wave (a) = 1.5 cm

Wavelength of the component wave (λ) = $\frac{2\pi}{k}$

$$= \frac{2\pi}{3\pi} = \frac{2}{3}\text{cm}$$

Velocity of the component wave (v) = $\frac{\omega}{k}$

$$\Rightarrow \quad v = \frac{30\pi}{3\pi} = 10 \text{ cm/s}$$

(ii) Distance between two adjacent nodes = $\frac{\lambda}{2}$

$$= \frac{1}{3}\text{cm}$$

(iii) In a stationary wave no beat is produced.

43. (i) What is the fundamental frequency of an organ pipe of length 75 cm, if the velocity of sound in air is 300 m/s?

(ii) Compare the fundamental frequency of an open organ pipe and a closed organ pipe of same length at same temperature.

(iii) Compare the length of an open organ pipe and a closed organ pipe if their fundamental frequency are same.

Sol. (i) Fundamental frequency = $\frac{v}{2L} = \frac{300 \times 100}{2 \times 75} = 200$ Hz

(ii) Fundamental frequency of an open organ pipe (ν) = $\frac{v}{2L}$

Fundamental frequency of a closed organ pipe (ν') = $\frac{v}{4L}$

$$\Rightarrow \quad \boxed{\nu = 2\nu'}$$

(iii) Fundamental frequency of an open organ pipe (ν) = $\frac{v}{2L}$

Fundamental frequency of an close organ pipe (ν') = $\frac{v}{4L'}$

Since we have $\nu = \nu'$

$\Rightarrow \quad \frac{v}{2L} = \frac{v}{4L'}$

$\Rightarrow \quad \boxed{L' = \frac{L}{2}}$

Length of the closed pipe is half the length of the open pipe.

44. A tuning fork produces 3 beats per second when it is sounded with a stretched string having tension either 25 or 16 newton. Calculate the frequency of the tuning fork.

Sol. Frequency of the wave produced in a string (ν) $\propto \sqrt{\text{Tension in the string}}$

$\Rightarrow \quad \nu = \sqrt{T}$

$$\frac{\nu_1}{\nu_2} = \sqrt{\frac{T_1}{T_2}}$$

$$\frac{\nu_1}{\nu_2} = \sqrt{\frac{25}{16}}$$

$$\nu_1 = \frac{5}{4}\nu_2$$

Let ν is the frequency of the fork, then according to the question, beat frequencies are

$\nu_1 - \nu = 3$

and $\nu - \nu_2 = 3$

$\Rightarrow \quad \nu_1 - \nu_2 = 6$

$$\frac{5}{4}\nu_2 - \nu_2 = 6$$

$$\frac{1}{4}\nu_2 = 6$$

ν_2 = 24 Hz

Therefore, ν_1 = $\frac{5}{4} \times 24 = 30$

Therefore, frequency of fork = 27 Hz

45. A man finds that the frequency of a stationary whistle decreases by 20% of its real frequency. Calculate the speed of the man along with its direction, if speed of sound in air is 330 m/s.

Sol. Let real frequency of the whistle = ν

$\therefore$ The observed frequency = ν – 20% of ν

$$= \frac{80}{100}\nu$$

$$= \frac{4}{5}\nu$$

Now in this case the source is at rest and the observer is moving. Since the apparent frequency decreases, the observer is moving away from the source.

$$\therefore \quad \nu' = \nu_0\left(\frac{v - v_0}{v}\right)$$

$$\frac{4}{5}\nu_0 = \nu_0\left(\frac{330 - v_0}{330}\right)$$

$$\frac{4}{5} = \left(\frac{330 - v_0}{330}\right)$$

$\Rightarrow \quad v_0 = 66$ m/s

Multiple Choice Questions

1. A tuning fork A produces 4 beats with tuning fork B of frequency 256 Hz. When A is filed beats are found to occur at shorter intervals. What was its original frequency of tuning fork A?
(a) 260 Hz (b) 252 Hz
(c) 256 Hz (d) 258 Hz

2. Two tuning forks A and B vibrating simultaneously produce 5 beats. Frequency of B is 512 Hz. It is seen that if one arm of A is filed, then the number of beats increases. The frequency of A will be
(a) 502 Hz (b) 507 Hz
(c) 517 Hz (d) 522 Hz

3. Two waves of length 50 cm and 51 cm produced 12 beats per second, The velocity of sound is
(a) 340 m/s (b) 331 m/s
(c) 306 m/s (d) 360 m/s

4. A set of 24 tuning forks are so arranged that each gives 6 beats per second with the previous one. If the frequency of the last tuning fork is double that of the first, frequency of the second tuning fork is
(a) 138 Hz (b) 144 Hz
(c) 132 Hz (d) 276 Hz

5. Two sitar strings A and B playing the note 'Ga' are slightly of tune and produce beats of frequency 6 Hz. The tension in the string A is slightly reduced and the beat frequency is found to reduce to 3 Hz. If the original frequency of A is 324 Hz, what is the frequency of B?
(a) 330 Hz (b) 318 Hz
(c) 324 Hz (d) 321 Hz

6. Two uniform wire are vibrating simultaneously in their fundamental notes. The tensions, length, diameters, and the densities of the two wire are in the ratio 8:1, 36:35, 4:1 and1:2 respectively. If the note of the higher pitch has a frequency 360 Hz, the number of beats produced per second is
(a) 5 (b) 10
(c) 15 (d) 20

7. The frequencies of two tuning forks A and B are respectively 1.5% more and 2.5% less than that of the tuning fork C. When A and B are sounded together, 12 beats are produced in 1 sec. The frequency of the tuning fork C is
(a) 200 Hz (b) 240 Hz
(c) 360 Hz (d) 300 Hz

8. The phenomenon of beats can take place
(a) for longitudinal waves only
(b) for transverse waves only
(c) for sound waves only
(d) for both longitudinal and transverse waves

9. A travelling wave is represented by the equation $y=\frac{1}{10}\sin\left(60t+2x\right)$, where x and y are in metres and t is in seconds. This represents a wave
(1) of frequency $\frac{30}{\pi}Hz$
(2) of wavelength π m
(3) of amplitude 10 cm
(4) moving in the positive x direction
Pick out the correct statements from the above.
(a) 1, 2, 4 (b) 1, 3, 4
(c) 1, 2, 3 (d) all

10. In a sinusoidal wave, the time required for a particular point ot move from maximum displacement to zero displacement is 0.170 s. The frequency of the wave is
(a) 1.47 Hz (b) 0.36 Hz
(c) 0.73 Hz (d) 2.94 Hz

11. The particles of a medium vibrate about their mean positions whenever a wave travels through that medium. The phase difference between the vibrations of two such particles
(a) varies with time
(b) varies with distance separating them
(c) varies with time as well as distance
(d) is always zero

12. The apparent frequency of a note is 200 Hz. When a listener is moving with a velocity of 40 m/s towards a stationary source. When he moves away from the same source, the apparent frequency of the same note is 160 Hz. The velocity of sound in air in m/s is
(a) 340 (b) 330
(c) 360 (d) 320

13. If a source of sound of frequency υ an a listener approach each other with a velocity equal to (1/20) of velocity of sound, the apparent frequency heard by the listener is
(a) $\left(\frac{21}{19}\right)\upsilon$ (b) $\left(\frac{20}{21}\right)\upsilon$

(c) $\left(\frac{21}{20}\right)\upsilon$ (d) $\left(\frac{19}{20}\right)\upsilon$

14. A sound is moving towards a stationary listener with 1/10th of the speed. The ratio of apparent to real frequency is

(a) $\frac{10}{9}$ (b) $\left(\frac{10}{9}\right)^2$

(c) $\left(\frac{11}{10}\right)^2$ (d) $\frac{11}{10}$

15. A whistle emitting a sound of frequency 440 Hz is tied to a string of 1.5 m length and rotated with an angular velocity of 20 rad/s in the horizontal plane. Then the range of frequencies heard by an observer stationed at a large distance from the whistle will be (Speed of sound v =330 m/s)
(a) 400.0 Hz to 484.0 Hz
(b) 403.3 Hz to 480.0 Hz
(c) 400.0 Hz to 480.0 Hz
(d) 403.3 Hz to 484.0 Hz

16. A car sounding its horn at 480 Hz moves towards a high wall at a speed of 20 m/s. If the speed of sound is 340 m/s, the frequency of the reflected sound heard by the man sitting in the car will be nearest to
(a) 480 Hz (b) 510 Hz
(c) 540 Hz (d) 570 Hz

17. A car is initially at rest, 330 m away from a stationary observer. It begins to move towards the observer with an acceleration of 1.1 ms^{-2}, sounding its horn continuously. 20 s later, the driver stops sounding the horn. The velocity of sound in air is 330 ms^{-1}. The observer sill hear the sound of the horn for a duration of
(a) 20 s (b) 21 s
(c) $20\frac{2}{3}s$ (d) $19\frac{1}{3}s$

18. A bat flies at a steady speed of 4 m s^{-1} emitting 90 kHz sound waves and is flying towards a wall. It detects a reflected signal at a frequency (speed of sound is 340 m s^{-1})
(a) 90.2 kHz (b) 91.2 kHz
(c) 92.2 kHz (d) 93.2 kHz

19. A source of sound producing wavelength 50 cm is moving away from a stationary observer with $\left(\frac{1}{5}\right)^{th}$ speed of sound. Then what is the wavelength of sound received by the observer?
(a) 55 cm (b) 40 cm
(c) 60 cm (d) 70 cm

20. Starting from the origin, a body oscillates simple harmonically with a period of 2s. After what time will its kinetic energy be 75% of the total energy?

(a) $\frac{1}{12}s$ (b) $\frac{1}{6}s$

(c) $\frac{1}{4}s$ (d) $\frac{1}{3}s$

21. A particle of mass m executes simple harmonic motion with amplitude A and frequency υ. The average kinetic energy during its motion from the position of equilibrium to the ends is
(a) $2\pi^2mA^2\upsilon^2$ (b) $\pi^2mA^2\upsilon^2$
(c) $\frac{1}{4}\pi^2mA^2\upsilon^2$ (d) $4\pi^2mA^2\upsilon^2$

22. The kinetic energy of a particle executing SHM will be equal to (1/8)th of its potential energy when its displacement from the mean position is (where A is the amplitude)

(a) $A\sqrt{2}$ (b) $\frac{A}{2}$

(c) $\frac{2\sqrt{2}}{3}A$ (d) $A\sqrt{\frac{2}{3}}$

23. When an oscillator completes 100 oscillations its amplitude is reduced to $\frac{1}{3}$ of initial value. What will be its amplitude, when it completes 200 oscillations?

(a) $\frac{1}{8}$ (b) $\frac{2}{3}$

(c) $\frac{1}{6}$ (d) $\frac{1}{9}$

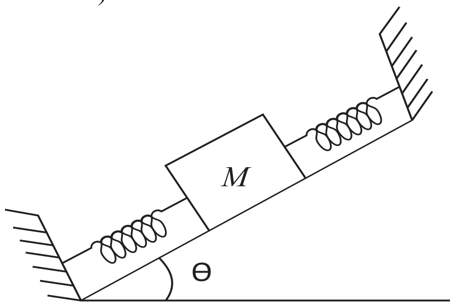
24. A particle oscillating under a force $\vec{F} = -k\vec{x} - b\vec{v}$ is a (k and b are constants)
(a) simple harmonic oscillator
(b) linear oscillator
(c) damped oscillator
(d) forced oscillator

25. Resonance is an example of
(a) forced oscillation
(b) damped oscillation
(c) free oscillation
(d) none of these

26. In case of oscillations of a body

(a) driving force is constant throughout
(b) driving force is to be applied only momentarily
(c) driving force has to be periodic and continuous
(d) driving force is not required

27. On a smooth inclined plane, a body of mass M is attached between two springs. The other ends of the springs are fixed to firm supports. If each spring has spring constant k, the period of oscillation of the body (assuming the spring as massless) is



(a) $2\pi\left(M/2k\right)^{1/2}$
(b) $2\pi\left(2M/k\right)^{1/2}$
(c) $2\pi\left(Ms\sin\theta/2k\right)$
(d) $2\pi\left(2Mg/k\right)^{1/2}$

28. A system of spring with their spring constants are as shown in figure. The frequency of oscillations of the mass m will (assuming the springs to be massless)

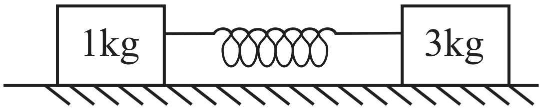
(a) $\dfrac{1}{2\pi}\sqrt{\dfrac{k_1k_2\left(k_3+k_4\right)}{\left[\left(k_1+k_2\right)+\left(k_3+k_4\right)+k_1k_4\right]m}}$

(b) $\dfrac{1}{2\pi}\sqrt{\dfrac{k_1k_2\left(k_3+k_4\right)}{\left[\left(k_1+k_2\right)+\left(k_3+k_4\right)+k_1k_2\right]n}}$

(c) $\dfrac{1}{2\pi}\sqrt{\dfrac{k_1k_2\left(k_3+k_4\right)}{\left[\left(k_1+k_2\right)\left(k_3+k_4\right)+k_1k_2\right]m}}$

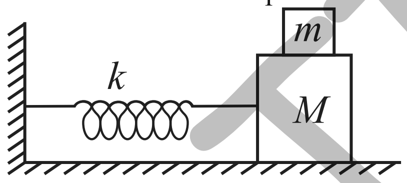
(d) $\dfrac{1}{2\pi}\sqrt{\dfrac{\left(k_1+k_2\right)\left(k_3+k_4\right)+k_1k_2}{k_1k_2\left(k_3+k_4\right)m}}$

29. Two masses of 3kg and 1kg are attached to apposite ends of a horizontal spring whose spring constant is 300 N/m as shown in figure. The natural vibrational frequency of the system is



(a) 4 Hz (b) 3 Hz
(c) 2 Hz (d) 1Hz

30. A mass M is attached to a horizontal spring of spring constant k fixed one side to rigid support as shown in figure. The mass oscillates on a frictionless surface with time period T and amplitude A. When the mass is in equilibrium position, another mass m is gently placed on it. What will be the new amplitude of oscillations?



(a) $A\sqrt{\dfrac{M}{M-m}}$ (b) $A\sqrt{\dfrac{M-m}{M}}$

(c) $A\sqrt{\dfrac{M}{M+m}}$ (d) $A\sqrt{\dfrac{M+m}{M}}$

31. A metal wire of length L_1 and area of cross section A is attached to a rigid support. Another metal wire of length L_2 and of the same cross sectional area is attached to the free end of the first wire. A body of mass M is then suspended from the free end of the second wire. If Y_1 and Y_2 are the Young's moduli of the wires respectively, the effective force constant of the system of two wires is

(a) $\dfrac{\left[\left(Y_1Y_2\right)A\right]}{\left[2\left(Y_1L_2+Y_2L_1\right)\right]}$

(b) $\dfrac{\left[\left(Y_1Y_2\right)A\right]}{\left(L_1L_2\right)^{1/2}}$

(c) $\dfrac{\left[\left(Y_1Y_2\right)A\right]}{\left(Y_1L_2+Y_2L_1\right)}$

(d) $\dfrac{\left[\left(Y_1Y_2\right)^{1/2}A\right]}{\left(L_1L_2\right)^{1/2}}$

32. If the frequency of human heart beat is 1.25 Hz, the number of heart beats in 1 minutes is
(a) 80 (b) 65
(c) 90 (d) 75

33. If two waves of the same frequency and amplitude respectively on superposition produce a resultant disturbance of the same amplitude the waves differ in phase by
(a) π (b) zero
(c) π/3 (d) 2π/3

34. Two sound waves travel in the same direction in a medium. The amplitude of each wave is A and

the phase difference between the two waves is 120°. The resultant amplitude will be
(a) $\sqrt{2}A$ (b) 2A
(c) 3A (d) A

35. Standing waves are produced by the superposition of two waves $y_1 = 0.05\ \sin(3\pi t - 2x)$ and $y_2 = 0.05\sin(3\pi t + 2x)$ where x and y are in metres and t is in second. What is the amplitude of the particle at x = 0.5 m? Given cos 57.3° = 0.54.
(a) 2.7 cm (b) 5.4 cm
(c) 8.1 cm (d) 10.8 cm

36. When a stationary wave is formed, then its frequency is
(a) same as that of the individual waves
(b) twice that of the individual waves
(c) half that of the individual waves
(d) $\sqrt{2}$ that of the individual waves

37. If the air column in a pipe which is closed at one end, is in resonance with a vibrating tuning fork at a frequency 260 Hz, then the length of the air column is
(a) 35.47 cm (b) 31.7cm
(c) 12.5 cm (d) 62.5 cm

38. An open and closed organ pipe have the same length. The ratio of p[th] mode of frequency of vibration of two pipes is
(a) 1 (b) P
(c) $p(2p+1)$ (d) $\dfrac{2p}{(2p-1)}$

39. Pick out the correct statement in the following with reference to stationary wave pattern.
(a) In a tube closed at one end, all the harmonics are present.
(b) In a tube open at one end, only even harmonics are present.
(c) The distance between successive nodes is equal to the wavelength.
(d) In a stretched string, the first overtone is the same as the second harmonic.

40. If the length of a stretched string is shortened by 40%, and the tension is increased by 44%, then the ratio of the final and initial fundamental frequencies is
(a) 3:4 (b) 4:3
(c) 1:3 (d) 2:1

41. An organ pipe closed at one end is excited to support the third overtone. It is found that air in the pipe has
(a) 3 nodes and 3 antinodes
(b) 3 nodes and 4 antinodes
(c) 4 nodes and 3 antinodes
(d) 4 nodes and 4 antinodes

42. Which of the following statements is wrong?
(a) In an open pipe the fundamental frequency is $\nu / 2L$.
(b) In a closed pipe the closed end is a displacement node.
(c) In an open pipe only the odd harmonics of fundamental frequency are present.
(d) In a closed pipe the fundamental frequency is $\nu / 4L$.

43. Regarding an open organ pipe, which of the following statement is correct?
(a) Both the ends are pressure antinodes.
(b) Both the ends are displacement nodes.
(c) Both ends are pressure nodes.
(d) None of these

44. Two uniform strings A and B made of steel are made to vibrate under the same tension. If the first overtone of A is equal to the second overtone of B and if the radius of A is twice that of B, the ratio of the lengths of the strings is
(a) 1:2 (b) 1:3
(c) 1:4 (d) 1:6

45. The second overtone of an open pipe has the same frequency as the first overtone of a closed pipe 2 m long. The length of the open pipe is
(a) 8 m (b) 4 m
(c) 2 m (d) 1 m

46. A glass tube of length 1.0 m is completely filled with water, A vibrating tuning fork of frequency 500 Hz is kept over the mouth of the tube and the water is drained out slowly at the bottom of the tube. If velocity of sound in air is 330 ms^{-1}, then the total number of resonances that occur will be
(a) 2 (b) 3
(c) 1 (d) 5

47. The third overtone of an open organ pipe of length L_o has the same frequency as the third overtone of a closed pipe of length L_c. The ratio L_o/L_c is equal to
(a) 2 (b) $\dfrac{3}{2}$
(c) $\dfrac{5}{3}$ (d) $\dfrac{8}{7}$

48. String A has a length L, radius of cross section r, density of material ρ and is under tension T.

String B has all these quantities double those of string A. If υ_A and υ_B are the corresponding fundamental frequencies of the vibrating string, then

(a) $\upsilon_A = 2\upsilon_B$ (b) $\upsilon_A = 4\upsilon_B$
(c) $\upsilon_B = 4\upsilon_A$ (d) $\upsilon_A = \upsilon_B$

49. A resonance pipe is open at both ends and 30 cm of its length is in resonance with an external frequency 1.1 kHz. If the speed of sound is 330 m/s which harmonic is in resonance

(a) first (b) second
(c) third (d) fourth

50. Two open organs pipes of fundamental frequencies υ_1 and υ_2 are joined in series. The fundamental frequency of the new pipe so obtained will be

(a) $\upsilon_1 + \upsilon_2$ (b) $\dfrac{\upsilon_1\upsilon_2}{(\upsilon_1 + \upsilon_2)}$
(c) $\dfrac{\upsilon_1\upsilon_2}{\upsilon_1 - \upsilon_2}$ (d) $\sqrt{(\upsilon_1^2 + \upsilon_2^2)}$

Answer Key

1. A	2. C	3. C
4. B	5. B	6. B
7. D	8. D	9. C
10. A	11. C	12. C
13. A	14. A	15. D
16. C	17. D	18. C
19. B	20. B	21. B
22. C	23. D	24. C
25. A	26. C	27. A
28. C	29. B	30. C
31. C	32. D	33. D
34. D	35. B	36. A
37. B	38. D	39. D
40. D	41. D	42. C
43. C	44. B	45. B
46. B	47. D	48. B
49. B	50. B	

JEE & NEET Previous Year

NEET & AIPMT

1. The fundamental frequency in an open organ pipe is equal to the third harmonic of a closed organ pipe. If the length of the closed organ pipe is 20 cm, the length of the open organ pipe is **[NEET 2018]**

(a) 12.5 cm (b) 8 cm
(c) 13.2 cm (d) 16 cm

2. A turing fork is used to produce resonance in a glass tube.The length of the air column in this tube can be adjusted by a variable piston.

At room temperature of $27^0 C$, two successive resonances are produced at 20 cm and 73 cm of column length.If the frequency of the tuning fork is 320 Hz, the velocity of sound in air at $27^0 C$. **[NEET 2017]**

(a) 350 m (b) 339 m
(c) 330 m (d) 300 m

3. The two nearest harmonics of a tube closed at one end and open at other end are 220 Hz and 260 Hz. What is the fundamental frequency of the system? **[NEET 2017]**

(a) 10 Hz (b) 20 Hz
(c) 30 Hz (d) 40 Hz

4. Two cars moving in opposite directions approach each other with speed of 22 m/s and 16.5 m/s respectively. The driver of the first car blows a horn having a frequency 400 Hz. The frequency heard by the driver of the second car is [velocity of sound 340 m/s] **[NEET 2017]**

(a) 350 Hz (b) 361 Hz
(c) 411 Hz (d) 448 Hz

5. A siren emitting a sound of frequency 800 Hz moves away from an observer towards a cliff at a speed of $15 ms^{-1}$. Then, the frequency of sound that the observer hears in the echo reflected from the cliff is (Take, velocity of sound in air = 330 ms^{-1}) **[NEET 2016]**

(a) 800 Hz (b) 838 Hz
(c) 885 Hz (d) 765 Hz

6. A uniform rope of length L and mass m_1 hangs vertically from a rigid support. A block of mass m_2 is attached to the free end of the rope. A transverse pulse of wavelength λ_1 is produced at the lower end of the rope. The wavelength of the pulse when it reaches the top of the rope is λ_2. The ratio λ_1/λ_2 **[NEET 2016]**

(a) $\sqrt{\frac{m_1+m_2}{m_2}}$ (b) $\sqrt{\frac{m_2}{m_1}}$
(c) $\sqrt{\frac{m_1+m_2}{m_1}}$ (d) $\sqrt{\frac{m_1}{m_2}}$

7. The second overtone of an open organ pipe has the same frequency as the first overtone of a closed pipe L metre long. The length of the open pipe will be **[NEET 2016]**

(a) L (b) $2L$
(c) $L/2$ (d) $4L$

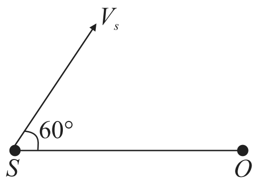
8. Three sound waves of equal amplitudes have frequencies $(n-1)$, n, $(n+1)$. They superimpose to give beats. The number of beats produced per second will be **[NEET 2016]**

(a) 1 (b) 4
(c) 3 (d) 2

9. The fundamental frequency of a closed organ pipe of length 20 cm is equal to the second overtone of an organ pipe open at both the ends. The length of organ pipe open at both the ends is **[AIPMT 2015]**

(a) 80 cm (b) 100 cm
(c) 120 cm (d) 140 cm

10. A source of sound S emitting waves of frequency 100 Hz and an observer O are located at some distance from each other. The source is moving with a speed of 19.4 ms^{-1} at an angle of 60° with the source observer line as shown in the figure. The observer is at rest. The apparent frequency observed by the observer (velocity of sound in air is 330 ms^{-1}), is **[AIPMT 2015]**



(a) 100 Hz (b) 103 Hz
(c) 106 Hz (d) 97 Hz

11. If n_1, n_2 and n_3 are the fundamental frequencies of three segments into which a string is divided, then the original fundamental frequency n of the string is given by **[AIPMT 2014]**

(a) $\frac{1}{n}=\frac{1}{n_1}+\frac{1}{n_2}+\frac{1}{n_3}$

(b) $\dfrac{1}{\sqrt{n}} = \dfrac{1}{\sqrt{n_1}} + \dfrac{1}{\sqrt{n_2}} + \dfrac{1}{\sqrt{n_3}}$

(c) $\sqrt{n} = \sqrt{n_1} + \sqrt{n_2} + \sqrt{n_3}$

(d) $n = n_1 + n_2 + n_3$

12. The number of possible natural oscillations of air column in a pipe closed at one end of length 85 cm whose frequencies lie below 1250 Hz are (velocity of sound = 340 ms^{-1}) **[AIPMT 2014]**

(a) 4 (b) 5
(c) 7 (d) 6

13. A speeding motorcyclist sees traffic jam ahead of him. He slows down to 36 km/h. He finds that traffic has eased and a car moving ahead of him at 18 km/h is honking at a frequency of 1392 Hz. If the speed of sound is 343 m/s, the frequency of the honk as heard by him will be **[AIPMT 2014]**

(a) 1332 Hz (b) 1372 Hz
(c) 1412 Hz (d) 1454 Hz

14. A wave travelling in the positive x-direction having displacement along y-direction as 1 m, wavelength 2π m and frequency of $\dfrac{1}{\pi}$ Hz is represented by **[NEET 2013]**

(a) $y = \sin(x - 2t)$
(b) $y = \sin(2px - 2pt)$
(c) $y = \sin(10px - 20pt)$
(d) $y = \sin(2px + 2pt)$

15. If we study the vibration of a pipe open at both ends, which of the following statements is not true? **[NEET 2013]**

(a) Open end will be antinode
(b) Odd harmonics of the fundamental frequency will be generated
(c) All harmonics of the fundamental frequency will be generated
(d) Pressure change will be maximum at both ends

16. A source of unknown frequency gives 4 beat/s when sounded with a source of known frequency 250 Hz. The second harmonic of the source of unknown frequency gives 5 heat/s when sounded with a source of frequency 513 Hz. The unknown frequency is **[NEET 2013]**

(a) 254 Hz (b) 246 Hz
(c) 240 Hz (d) 260 Hz

17. When a string is divided into three segments of lengths l_1, l_2 and l_3, the fundamental frequencies of these three segments are v_1, v_2 and v_3 respectively. The original fundamental frequency (v) of the string is **[AIPMT 2012]**

(a) $\sqrt{v} = \sqrt{v_1} + \sqrt{v_2} + \sqrt{v_3}$

(b) $v = v_1 + v_2 + v_3$

(c) $\dfrac{1}{v} = \dfrac{1}{v_1} + \dfrac{1}{v_2} + \dfrac{1}{v_3}$

(d) $\dfrac{1}{\sqrt{v}} = \dfrac{1}{\sqrt{v_1}} + \dfrac{1}{\sqrt{v_2}} + \dfrac{1}{\sqrt{v_3}}$

18. Two sources of sound placed closed to each other, are emitting progressive waves given by $y_1 = 4 \sin 600pt$ and $y_2 = 5 \sin 608pt$. An observer located near these two sources of sound will hear **[AIPMT 2012]**

(a) 4 beat/s with intensity ratio 25 : 16 between waxing and waning
(b) 8 beat/s with intensity ratio 25 : 16 between waxing and waning
(c) 8 beat/s with intensity ratio 81 : 1 between waxing and waning
(d) 4 beat/s with intensity ratio 81 : 1 between waxing and waning

19. Two waves are represented by the equations $y_1 = \alpha \sin (wt + kx + 0.57)$ m and $y_2 = \alpha \cos (wt + kx)$m, where x is in metre and t in second. The phase difference between them is **[AIPMT 2011]**

(a) 1.25 rad (b) 1.57 rad
(c) 0.57 rad (d) 1 rad

20. Sound waves travel at 350 m/s through a warm air and at 3500 m/s through brass. The wavelength of a 700 Hz acoustic wave as it enters brass from warm air **[AIPMT 2011]**

(a) increases by a factor 20
(b) increases by a factor 10
(c) decreases by a factor 20
(d) decreases by a factor 10

21. A transverse wave is represented by $y = A \sin (wt - kx)$. For what value of the wavelength is the wave velocity equal to the maximum particle velocity? **[AIPMT 2010]**

(a) $\pi A/2$ (b) ωA
(c) $2\pi A$ (d) A

22. A tuning fork of frequency 512 Hz makes 4 beat/s with the vibrating string of a piano. The beat frequency decreases to 2 beat/s when the tension in the piano string is slightly increased. The frequency of the piano string before increasing the tension was **[AIPMT 2010]**

(a) 510 Hz (b) 514 Hz
(c) 516 Hz (d) 508 Hz

23. The driver of a car travelling with speed 30 ms^{-1}

towards a hill sounds a horn of frequency 600 Hz. If the velocity of sound in air is 330 ms^{-1}, the frequency of reflected sound as heard by driver is **[AIPMT 2009]**
(a) 550 Hz (b) 555.5 Hz
(c) 720 Hz (d) 500 Hz

24. A wave in a string has an amplitude of 2cm. The wave travels in the positive direction of x-axis with a speed of 128 ms^{-1} and it is noted that 5 complete waves fit in 4 m length of the string. The equation describing the wave is **[AIPMT 2009]**
(a) $y = (0.02)$m $\sin(7.85 \times + 1005t)$
(b) $y = (0.02)$m $\sin(15.7 \times - 2010t)$
(c) $y = (0.02)$m $\sin(15.7 \times +2010t)$
(d) $y = (0.02)$ m $\sin(7.85 \times - 1005t)$

25. The wave described by
$y = 0.25\sin(10\ \text{p}x - 2\text{p}t)$,
where, x and y are in metre and t in second, is a wave travelling along the **[AIPMT 2008]**
(a) negative x-direction with frequency 1 Hz
(b) positive x-direction with frequency π Hz and wavelength $\lambda = 0.2$ m
(c) positive x-direction with frequency 1 Hz and wavelength $\lambda = 0.2$ m
(d) negative x-direction with amplitude 0.25 m and wavelength $\lambda = 0.2$ m

26. Two periodic waves of intensities I_1 and I_2 pass through a region at the same time in the same direction. The sum of the maximum and minimum intensities is **[AIPMT 2008]**
(a) $I_1 + I_2$ (b) $(\sqrt{I_1} + \sqrt{I_2})^2$
(c) $(\sqrt{I_1} - \sqrt{I_2})^2$ (d) $2(I_1 + I_2)$

27. Which one of the following statements is true? **[AIPMT 2006]**
(a) Both light and sound waves in air are transverse
(b) The sound waves in air are longitudinal while fhe light waves are transverse
(c) both light and sound waves in air are longitudinal
(d) Both light and sound waves can travel in vacuum

28. The time of reverberation of a room A is 1s. What will be the time (in second) of reverberation of a room, having all the dimensions double of those of room A? **[AIPMT 2006]**
(a) 2 (b) 4
(c) $\frac{1}{2}$ (d) 1

29. A transverse wave propagating along x-axis is represented by $y(x, t) = 8\sin\left(0.5\pi x - 4\pi t - \frac{\pi}{4}\right)$. where, x is in metre and t is in second. The speed of the wave is **[AIPMT 2006]**
(a) 4π m/s (b) 0.5π m/s
(c) $\frac{\pi}{4}$ m/s (d) 8 m/s

30. Two sound waves with wavelengths 5 m and 5.5 m respectively, each propagate in a gas with velocity 330 m/s. We expect the following number of beat per second **[AIPMT 2006]**
(a) 12 (b) zero
(c) 1 (d) 6

31. Two vibrating tuning forks produce progressive waves given by $y_1 = 4\sin 500\ \text{p}t$ and $y_2 = 2\sin 506\ \text{p}t$. Number of beat produced per minute is **[AIPMT 2005]**
(a) 360 (b) 180
(c) 3 (d) 60

32. A point source emits sound equally in all directions in a non-absorbing medium. Two points P and Q are at distance of 2m and 3m respectively from the source. The ratio of the intensities of the waves at P and Q is **[AIPMT 2005]**
(a) 9 : 4 (b) 2 : 3
(c) 3 : 2 (d) 4 : 9

33. The phase difference between two waves, represented by

$$y_1 = 10^{-6}\sin\left\{100\,t + \left(\frac{x}{50}\right) + 0.5\right\}\text{m}$$

$$y_2 = 10^{-6}\cos\left\{100\,t + \left(\frac{x}{50}\right)\right\}\text{m},$$

where, x is expressed in metre and t is expressed in second, is approximately **[AIPMT 2004]**
(a) 1.07 rad (b) 2.07 rad
(c) 0.5 rad (d) 1.5 rad

34. A car is moving towards a high cliff. The car driver sounds a horn of frequency f. The reflected sound heard by the driver has a frequency $2f$. If v be the velocity of sound, then the velocity of the car, in the same velocity units, will be **[AIPMT 2004]**
(a) $\frac{v}{\sqrt{2}}$ (b) $\frac{v}{3}$
(c) $\frac{v}{4}$ (d) $\frac{v}{2}$

35. An observer moves towards a stationary source of sound with a speed $\frac{1}{5}$th of the speed of sound. The wavelength and frequency of the source emitted are λ and f respectively. The apparent frequency and wavelength recorded by the observer are respectively **[AIPMT 2003]**

(a) f, 1.2 λ. (b) 0.8f, 0.8l
(c) 1.2f, 1.2 λ (d) 1.2f, λ.

36. A whistle revolves in a circle with angular velocity ω = 20 rad/s using a string of length 50 cm. If the actual frequency of sound from the whistle is 385 Hz, then the minimum frequency heard by the observer far away from the centre is (velocity of sound v = 340 m/s) **[AIPMT 2002]**

(a) 385 Hz (b) 374 Hz
(c) 394 Hz (d) 333 Hz

37. A wave of amplitude a = 0.2 m, velocity v = 360 m/s and wavelength 60 m is travelling along positive x-axis, then the correct expression for the wave is **[AIPMT 2002]**

(a) $y = 0.2 \sin 2\pi\left(6t + \frac{x}{60}\right)$

(b) $y = 0.2 \sin \pi\left(6t + \frac{x}{60}\right)$

(c) $y = 0.2 \sin 2\pi\left(6f - \frac{x}{60}\right)$

(d) $y = 0.2 \sin \pi\left(6f - \frac{x}{60}\right)$

38. The equation of a wave is given by $y = a \sin\left(100t - \frac{x}{10}\right)$, where x and y are in metre and t in second, then velocity of wave is **[AIPMT 2001]**

(a) 0.1 m/s (b) 10 m/s
(c) 100 m/s (d) 1000 m/s

39. A wave enters to water from air. In air frequency, wavelength, intensity and velocity are n_1, l_1, I_1 and v_1 respectively. In water the corresponding quantities are n_2, l_2, I_2 and v_2 respectively, then **[AIPMT 2001]**

(a) $I_1 = I_2$ (b) $n_1 = n_2$
(c) $v_1 = v_2$ (d) $l_1 = l_2$

40. Equations of two progressive waves are given by $y_1 = a \sin (wt + f_1)$ and $y_2 = a \sin (wt + f_2)$. If amplitude and time period of resultant wave are same as that of both the waves, then $(f_1 - f_2)$ is **[AIPMT 2001]**

(a) $\frac{\pi}{3}$ (b) $\frac{2\pi}{3}$
(c) $\frac{\pi}{3}$ (d) $\frac{\pi}{4}$

41. Two strings A and B have lengths I_A and I_B and carry masses M_A and M_B at their lower ends, the upper ends being supported by rigid supports. If n_A and n_B are the frequencies of their vibrations and $n_A = 2\, n_B$, then **[AIPMT 2000]**

(a) $I_A = 4I_B$, regardless of masses
(b) $I_B = 4I_A$, regardless of masses
(c) $M_A = 2M_B$, $I_A = 2I_B$
(d) $M_B = 2$, $M_A.I_B = 2I_A$

42. A sonometer wire when vibrated in full length has frequency n. Now, it is divided by the help of bridges into a number of segments of lengths l_1, l_2, l_3.... When vibrated these segments have frequencies n_1, n_2, n_3… Then, the correct relation is **[AIPMT 2000]**

(a) $n = n_1 + n_2 + n_3 + ...$
(b) $n^2 = n_1^2 + n_2^2 + n_3^2 + ...$
(c) $\frac{1}{n} = \frac{1}{n_1} + \frac{1}{n_2} + \frac{1}{n_3} + ...$
(d) $\frac{1}{\sqrt{n}} = \frac{1}{\sqrt{n_1}} + \frac{1}{\sqrt{n_2}} + \frac{1}{\sqrt{n_3}}$

43. Two sources are at a finite distance apart. They emit sounds of wavelength λ. An observer situated between them on line joining approaches one source with speed u. Then, the number of beat heard/second by observer will be **[AIPMT 2000]**

(a) $\frac{2u}{\lambda}$ (b) $\frac{u}{\lambda}$
(c) $\frac{u}{2\lambda}$ (d) $\frac{\lambda}{u}$

JEE & AIEEE

1. A stationary observer sound from two identical tuning forks, one of which approaches and the other one recedes with the same speed (much less than the speed of sound). The observer hears 2 beats/sec. The oscillation frequency of each tuning fork is 0 = 1400 Hz and the velocity of sound in air is 350 m/s. The speed of each tuning fork is close to **[JEE Mains 2020]**

(a) $\frac{1}{4}$ m/s (b) $\frac{1}{2}$ m/s
(c) 1 m/s (d) $\frac{1}{8}$ m/s

2. Length of a string tied to two rigid supports is 40 cm. Maximum length (wave length in cm) of stationary wave produced on it is **[AIEEE 2002]**

(a) 20 (b) 80
(c) 40 (d) 120

3. A tuning fork of known frequency 256 Hz makes 5 beats per second with the vibrating string of a piano. The beat frequency decreases to 2 beats per second when the tension in the piano string is slightly increased. The frequency of the piano string before increasing the tension was **[AIEEE 2003]**

(a) (256+ 2) Hz (c) (256–5) Hz
(b) (256–2) Hz (d) (256+ 5) Hz

4. The displacement y of a particle in a medium can be expressed as $y = 10^{-6}\sin(100t + 20x + \pi/4)$m, where $t$is in second and $x$in meter. The speed of the wave is **[AIEEE 2004]**

(a) 2000 m/s (b) 5 m/s
(c) 20 m/s (d) 5 pm/s

5. whentwotuning forks (fork 1 and fork 2) are sounded simultaneously, 4 beats per second are heard. Now some tape is attached on the prong of the fork 2. When the tuning forks are sounded again, 6 beats per second are heard. If the frequency of fork 1 is 200 Hz, then what was the original frequency of fork 2? **[AIEEE 2005]**

(a) 200 Hz (b) 202 Hz
(c) 196 Hz (d) 2014 Hz

6. A string is stretched between fixed points separated by 75 cm. It is observed to have resonant frequencies of 420 Hz and 315 Hz. There are no other resonant frequencies between these two. Then, the lowest resonant frequency for this string is **[AIEEE 2006]**

(a) 10.5 Hz (c) 1.05 Hz
(b) 105 Hz (d) 1050 Hz

7. A wave travelling along the x–axis is described by the equation $y(x, t) = 0.005\cos(ax - bt)$. If the wavelength and the time period of the wave are 0.08 m and 2.0 s, respectively, then α and β (in appropriate units are) **[AIEEE 2008]**

(a) $\alpha = 25.00\,\pi, \beta = p$

(b) $\alpha = \frac{0.08}{\pi}, \frac{2.0}{\pi}$

(c) $\alpha = \frac{0.04}{\pi}, \beta\,\frac{1.0}{\pi}$

(d) $\alpha = 12.50\,\pi, \beta = \frac{\pi}{2.0}$

8. Three sound waves of equal amplitudes have frequencies $(f - 1)$, f and $(f + 1)$. The superpose to give beats. The number of beats produced per second will be **[AIEEE 2008]**

(a) 4 (b) 3
(c) 2 (d) 1

9. The equation of a wave on a string of linear mass density 0.04 kg m^{-1} is given by

$$y = 0.02\ (\text{m})\ \sin\left[3\pi\left(\frac{t}{0.04(\text{s})} - \frac{x}{0.50(\text{m})}\right)\right]$$

The tension in the string is **[AIEEE 2010]**

(a) 6.25 N (b) 4.0 N
(c) 12.5 N (d) 0.5 N

10. The transverse displacement $y(x, t)$ of a wave on a string is given by $y(x, t) = e^{-(ax^2 + bt^2 + 2\sqrt{ab}xt)}$. This represents a **[AIEEE 2011]**

(a) wave moving in $-x$ direction with speed $\sqrt{\frac{b}{a}}$.

(b) standing wave of frequency $\sqrt{b}$.

(c) standing wave of frequency $\frac{1}{\sqrt{b}}$.

(d) wave moving in $+x$ direction with speed $\sqrt{\frac{a}{b}}$.

11. A cylindrical tube, open at both ends, has a fundamental frequency, f, in air. The tube is dipped vertically in water so that half of it is in water. The fundamental frequency of the air-column is now **[JEE Mains 2015]**

(a) f (b) $\frac{f}{2}$

(c) $\frac{3f}{4}$ (d) $2f$

12. An observer is moving with half the speed of light towards a stationary microwave source emitting waves at frequency 10 GHz. What is the frequency of the microwave measured by the observer? (Speed of light = 3×10^8m s^{-1}) **[JEE Mains 2017]**

(a) 10.1 GHz (b) 12.1GHz
(c) 17.3 GHz (d) 15.3 GHz

13. A standing was is formed by the superposition of two waves travelling in opposite directions. The transverse displacement is given by

$$y(x, t) = 0.5\sin\left(\frac{5\pi}{4}x\right)\cos(200\,pt)$$

What is the speed of the traveling wave moving in the positive x and t are in metre the second, respectively) **[JEE Mains 2017]**

(a) 90 m/s (b) 180 m/s

(c) 120 m/s (d) 160 m/s

NEET & AIPMT Answer Key

1. C	2. B	3. B
4. D	5. B	6. A
7. B	8. A	9. C
10. B	11. A	12. D
13. C	14. A	15. D
16. A	17. C	18. D
19. D	20. B	21. C
22. D	23. C	24. D
25. C	26. D	27. B
28. A	29. D	30. D
31. B	32. A	33. A
34. B	35. D	36. B
37. C	38. D	39. B
40. B	41. B	42. C
43. A		

JEE Answer Key

1. A	2. B	3. C
4. B	5. D	6. B
7. A	8. D	9. A
10. A	11. A	12. D
13. D		